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NGA STANDARDIZATION DOCUMENT

Frame Sensor Model
Metadata Profile
Supporting Precise Geopositioning
(2011-07-07)

Version 2.1

NATIONAL CENTER FOR GEOSPATIAL INTELLIGENCE STANDARDS

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Revision History

Version Identifier	Date	Revisions/notes
xx.1	16 March 2009	Mikhail comments and review of pushbroom metadata
xx.1	25 March 2009	Clean up of line for line review by Mikhail on 17 March
xx.1	30 March 2009 and 2 Apr 2009	Clean up based on Mikhail review provided 3/29 and 3/30/09
Almost 2	5 Apr 2009	Almost Final edits and format cleanup
2	21 July 2009	Final rework based on MISB 0801 and SENSRB crosswalk edits, Error Prop appendix development and consolidation of transformations to a single matrix
2.1	07 July 2011	Version 2.0 revision necessary to correct Equation 2’s formula and reformatting of Appendix A Equation numbering.

1. Introduction

1.1 Background/Scope

The National Geospatial-Intelligence Agency (NGA) in coordination with the Department of Defense (DoD) components and the Intelligence Community (IC) have cooperated to standardize descriptions of the essential data characteristics necessary for precise positioning from a variety of sensor collection systems by creating "sensor models". These information/guidance primers describe in detail the information about the physics and dynamics of the specific collection system required to implement photogrammetry equations to establish the geometric relationship between the image and object imaged. This is the first of the set of primers and also serves as the basic template for establishing similar documents for other types of sensors, e.g., synthetic aperture radar (SAR), pushbroom, whiskbroom, LIDAR, etc. This document will enable the validation and Configuration Management (CM) of geopositioning capabilities across the National System for Geospatial Intelligence (NSG) and serve as a design document for the DoD and IC Acquisition community.

1.2 Approach

This technical document defines a data model of the optimal minimum set of parameters required to mathematically establish the physical relationship between a sensor's image record and an object of interest when employing classic photogrammetric equations. If the optimal set of parameters are not directly available, alternative platform and sensor parameters are identified that can be employed to establish the preferred set. A frame sensor is one that acquires all of the picture element (pixel) data for a single area image record (frame) at an instant of time. Typical of this class of sensor is that it has a fixed exposure and is comprised of a two-dimensional detector or array, e.g., focal plane array (FPA) or Charge-Coupled Device (CCD) array.

"Sensor" usually refers to digital collections; the term "camera", if used, is typically used to denote use of film-based collectors. The focus of this report will be on those geometric sensor properties necessary for accurate and precise geolocation with electro-optical (visible) frame sensors and not on the spectral sensitivity of the sensor; although the definitions and development apply equally to film and infrared (IR) arrays.

1.3 Normative References

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO TC/211 211n1197, 19101 Geographic information – Reference model, as sent to the ISO Central Secretariat for registration as FDIS, December 3, 2001.

ISO TC/211 211n2047, Text for ISO 19111 Geographic Information - Spatial referencing by coordinates, as sent to the ISO Central Secretariat for issuing as FDIS, July 17, 2006.

ISO TC/211 211n2171, Text for final CD 19115-2, Geographic information - Metadata - Part 2: Extensions for imagery and gridded data, March 8, 2007.

ISO TC211 211n1017, Draft review summary from stage 0 of project 19124, Geographic information - Imagery and gridded data components, December 1, 2000.

ISO TC211 211n1869, New Work Item proposal and PDTS 19129 Geographic information - Imagery, gridded and coverage data framework, July 14, 2005.

Federal Geographic Data Committee (FGDC) Document Number FGDC-STD-012-2002, Content Standard for Digital Geospatial Metadata: Extensions for Remote Sensing Metadata.

Open Geospatial Consortium Inc. Transducer Markup Language Implementation Specification, Version 1.0.0, OGC® 06-010r6, December 22, 2006.

Open Geospatial Consortium Inc. Sensor Model Language (SensorML) Implementation Specification, Version 1.0, OGC® 07-000, February 27, 2007.

Community Sensor Model (CSM) Technical Requirements Document, Version 3.0, December 15, 2005.

North Atlantic Treaty Organization (NATO) Standardization Agreement (STANAG), Air Reconnaissance Primary Imagery Data Standard, Base document STANAG 7023 Edition 3, June 29, 2005.

National Geospatial-Intelligence Agency. National Imagery Transmission Format Version 2.1 For The National Imagery Transmission Format Standard, MIL-STD-2500C, May 1, 2006.

National Imagery and Mapping Agency. System Generic Model, Part 5, Generic Sensors, December 16, 1996.

Mikhail, Edward M., James S. Bethel, and J. Chris McGlone. Introduction to Modern Photogrammetry. New York: John Wiley & Sons, Inc, 2001.

NGA Motion Imagery Standards Board (MISB) Engineering Guidance 0801, Nov 2008

Stevens, B., Lewis, F. Aircraft Control and Simulation, 2nd Edition. Wiley-Interscience; 2nd edition, October 6, 2003

1.4 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

1.4.1. adjustable model parameters

model parameters that can be refined using available additional information such as ground control points, to improve or enhance modelling corrections

1.4.2. area recording

"instantaneously" recording an image in a single frame

1.4.3. attitude

orientation of a body, described by the angles between the axes of that body's **coordinate system** and the axes of an external **coordinate system** [ISO 19116]

1.4.4. attribute

named property of an entity [ISO/IEC 2382-17]

1.4.5. calibrated focal length

distance between the **projection center** and the **image plane** that is the result of balancing positive and negative radial lens distortions during sensor calibration

1.4.6. coordinate

one of a sequence of *n* numbers designating the position of a point in *n*-dimensional space [ISO 19111]

NOTE: In **a coordinate reference system**, the numbers must be qualified by units.

1.4.7. coordinate reference system

coordinate system that is related to the real world by a datum [ISO 19111]

NOTE: For geodetic and vertical datums, it will be related to the Earth.

1.4.8. coordinate system

set of mathematical rules for specifying how **coordinates** are to be assigned to points [ISO 19111]

1.4.9. data

reinterpretable representation of information in a formalised manner suitable for communication, interpretation, or processing [ISO/IEC 2382-1]

1.4.10. error propagation

determination of the covariances of calculated quantities from the input covariances of known values

1.4.11. fiducial center

point determined on the basis of the camera fiducial marks.

NOTE: When there are four fiducial marks, it is the intersection of the two lines connecting the pairs of opposite fiducials.

1.4.12. fiducial mark

one of four or more marks attached to the frame of a camera that are assigned **coordinates** when the camera is calibrated

NOTE: The **fiducial marks** are mechanical devices, which are firmly attached to the frame of the camera. During the exposure, the film is pressed from the back against the frame and the **fiducial marks**. The negatives of the **fiducial marks** appear on the exposed film. The **fiducial marks** allow for the establishment of an image coordinate system and their calibrated values assist in correcting for film distortion.

1.4.13. frame sensor

sensor that collects all of the picture element (pixel) data for a single area image record (frame) at an instant of time

1.4.14. geodetic coordinate system

coordinate system in which position is specified by geodetic latitude, geodetic longitude and (in the three-dimensional case) ellipsoidal height [ISO 19111]

1.4.15. geodetic datum

datum describing the relationship of a **coordinate system** to the Earth [ISO 19111]

NOTE 1: In most cases, the **geodetic datum** includes an ellipsoid description

NOTE 2: The term and this Technical Specification may be applicable to some other celestial bodies.

1.4.16. geographic information

information concerning phenomena implicitly or explicitly associated with a location relative to the Earth [ISO 19101]

1.4.17. geographic location

longitude, latitude and elevation of a ground or elevated point

1.4.18. geolocating

geopositioning an object using a sensor model

1.4.19. geopositioning

determining the ground coordinates of an object from image coordinates

1.4.20. ground control point

point on the ground that has accurately known geographic location

1.4.21. image

coverage whose attribute values are a numerical representation of a remotely sensed physical parameter

NOTE: The physical parameters are the result of measurement by a **sensor** or a prediction from a model.

1.4.22. image coordinates

coordinates with respect to a Cartesian coordinate system of an image

NOTE: The image coordinates can be in pixel or in a measure of length (linear measure).

1.4.23. image distortion

deviation in the location of an actual image point from its theoretically correct position according to the geometry of the imaging process

1.4.24. image-identifiable ground control point

ground control point associated with a marker or other object on the ground that can be recognized in an **image**

NOTE: The ground control point may be marked in the image, or the user may be provided with an unambiguous description of the ground control point so that it can be found in the image.

1.4.25. image plane

plane behind an imaging lens where images of objects within the depth of field of the lens are in focus

1.4.26. image point

point on the **image** that uniquely represents an **object point**

1.4.27. image record

see **image** ? the result of processing the energized detector(s) into a visible spectrum image

1.4.28. imagery

representation of objects and phenomena as sensed or detected (by camera, infrared and multispectral scanners, radar and photometers) and of objects as **images** through electronic and optical techniques [19101-2]

1.4.29. metadata

data about **data** [ISO 19115]

1.4.30. nodal point

in optics, a point at the center of the lens

1.4.31. object point

point in the object space that is imaged by a **sensor**

NOTE: In **remote sensing** and aerial photogrammetry an **object point** is a point defined in the ground **coordinate reference system**.

1.4.32. objective

optical element that receives light from the object and forms the first or primary **image** of an optical system

1.4.33. passive sensor

sensor that detects and collects energy that already exists (such as reflected energy from the Sun)

1.4.34. platform coordinate reference system

coordinate reference system fixed to the collection platform within which positions on the collection platform are defined

1.4.35. pixel

picture element [ISO 19101-2]

1.4.36. principal point of autocollimation

point of intersection between the **image plane** and the normal from the **projection center**

1.4.37. projection center

point located in three dimensions through which all rays between **object points** and **image points appear to pass geometrically**.

NOTE: It is represented by the rear nodal point of the imaging lens system.

1.4.38. remote sensing

collection and interpretation of information about an object without being in physical contact with the object

1.4.39. sensor

element of a measuring instrument or measuring chain that is directly affected by the measurand [ISO 19101-2]

1.4.40. sensor model

mathematical description of the relationship between the three-dimensional object space and the associated two-dimensional image plane

1.5 Symbols and abbreviated terms

1.5.1 Abbreviated terms

ASC	Aeronautical Systems Center
API	Application Program Interface
CCD	Charge-Coupled Device
CCS	Common Coordinate System
CM	Configuration Management
COTS	Commercial Off-The-Shelf
CSMS	Community Sensor Model Standard
CSMWG	Community Sensor Model Working Group
D	Down
DCGS	Distributed Common Ground/Surface System
DoD	Department of Defense
ECEF	Earth-centered, Earth-fixed
EG	Engineering Guideline
EGM	Earth Gravity Model
ENU	East-North-Up
EO	Exterior Orientation
FPA	Focal Plane Array
FR&T	Future Requirements and Technologies
GEOTRANS	Geographic Translator
GPS	Global Positioning System
GWG	Geospatial Intelligence Standards Working Group
IMINT	Imagery Intelligence
INS	Inertial Navigation System
IR	Infrared
ISO	International Organization for Standardization
ITS	Information Technology Standards Committee
MISB	Motion Imagery Standards Board
MSL	Mean Sea Level
NATO	North Atlantic Treaty Organization
NED	North-East-Down
NGA	National Geospatial-Intelligence Agency (former NIMA)

NIMA	National Imagery and Mapping Agency
NITF	National Imagery Transmission Format
NCDM	National System for Geospatial Intelligence Conceptual Data Model
RSM	Replacement Sensor Model
S^2AG	Sensor Standards Acquisition Guide
SAR	Synthetic Aperture Radar
SensorML	Sensor Markup Language
STANAG	Standardization Agreement (NATO)
TCPED	Tasking, Collection, Processing, Exploitation and Dissemination
TML	Transducer Markup Language
TRE	Tagged Record Extension
UTC	Coordinated Universal Time
WGS	World Geodetic System

1.5.2 Symbols

A	object, or ground point
a	image point
a_1, a_2, b_1, b_2	scale and skew coefficients
b_x, b_y, b_z	base vector (lever arm) GPS to sensor projection center components
c_1, c_2	translation shift scalars
c	pixel column number (may be fractional)
C	number of columns (samples) on the collection array (unitless)
C_s	linear translations from CCS to collection array y-axis
C_ℓ	linear translations from CCS to collection array x-axis
d_x	pixel width, mm
d_y	pixel height, mm
E	East
f	sensor focal length, mm
g	Geocentric
H	sensor altitude, m HAE; also platform Heading
HAE	height above ellipsoid
H_{msl}	sensor altitude, km MSL
h_{msl}	object elevation, km MSL
h	object elevation, m HAE
J	Jacobean
K	refraction constant, micro-radians
k	arbitrary constant, unique scale factor per ground point
k_0, k_1, k_2, k_3	base, first, second, and third order radial distortion coefficients, respectively
km	kilometer
L	sensor perspective center
ℓ	line
M	orientation (rotation) matrix
N	North
n	NED
P	Platform pitch
p	platform
p_1, p_2	decentering coefficients
r	radial distance, image record, pixel row number (may be fractional)
R	number of rows (lines) on the collection array (unitless); Platform Roll
s	sample, sensor
U	Up
X	X coordinates within Earth coordinate system

X_L	Platform longitudinal axis
x	various x coordinates defined by subscripts, collection array coordinate reference system x-axis, coordinate
x_0	x-coordinate of the foot of the perpendicular dropped from perspective center of the camera lens (mm) to the image plane
x'	corrected image x coordinate
x_r	sensor coordinate reference system x-axis, at the sensor perspective center
Y	Y coordinates within Earth coordinate system
Y_L	Platform pitch axis
y	various y coordinates defined by subscripts, collection array coordinate reference system y-axis, coordinate
y_0	y-coordinate of the foot of the perpendicular dropped from sensor perspective center of the camera lens (mm) to the image plane
y'	corrected image y coordinate
y_r	sensor coordinate reference system y-axis, at the lens center or perspective center axis
Z	Z coordinates within Earth coordinate system
Z_L	Platform yaw axis
z	various z coordinates, defined by subscripts
z_r	sensor coordinate reference system z-axis, at the lens center or perspective center axis
α	angle the refracted ray makes with local vertical
δr	radial optical distortion
Δd	atmospheric refraction angular displacement
Δx_{decen}	rotational symmetry, decentering, x component
Δy_{decen}	rotational symmetry, decentering, y-component
Δx_{lens}	total lens radial distortion and decentering, x-component
Δy_{lens}	total lens radial distortion and decentering, x-component
Δx_{radial}	atmospheric refraction, x-component
Δy_{radial}	atmospheric refraction, y-component
Δx_{ref}	radial optical distortion x-component
Δy_{ref}	radial optical distortion y-component
ϕ	latitude, pitch
λ	longitude
κ	yaw
σ	covariance matrix element
ω	roll
$\sum$	covariance matrix

2. Overview for Coordinate System Descriptions and Relationships

2.1 General Coordinate Reference System Considerations

A mathematical relationship exists between the position of an object on the earth's surface and its image as recorded by an overhead sensor. The objective is to coherently describe that relationship so that it can be used by image exploitation systems and applications.

Typically, an image's spatial position will be given in relation to a coordinate system locally defined or attached to the sensor. Likewise, the corresponding object's position will be defined with respect to a coordinate system attached to an earth-based datum. Therefore, with the assumption that both of the coordinate systems in use are orthogonal, the transformation from a sensor-based coordinate system to an earth-based coordinate system is accomplished via a sequence of translations and rotations of the sensor's coordinate system origin and axes until the sensor coordinate system coincides with the earth-based coordinate system's origin and axes.

The sensor position may be provided directly or require derivation based on any number of intermediary coordinate systems with emphasis placed on those of an aerial platform. There may be one or more gimbals to which the sensor is attached, each with its own coordinate system. In addition, the platform's typical positional reference to the Global Positioning System (GPS) and on-board inertial navigation system (INS) are physically offset from the sensor. Transforming between each coordinate system into the common frames of reference of the image and earth-centered coordinates is incorporated into the mathematical model of the frame sensor. An example case is addressed in Appendix A.

Airborne platforms normally employ GPS and INS systems to define position and attitude. Except for systems developed since 2006, the GPS antenna and the INS gyros and accelerometers typically were not physically embedded with the sensor as illustrated in Figure 1.

For a GPS receiver, all observations are at the phase center of the antenna. The analogous point for an Inertial Measurement Unit (IMU) is the intersection of the three sensitivity axes. The physical offset between the two generally is termed the 'lever-arm'. Denoting the 'lever-arm' vector from the GPS antenna phase center to the INS is the vector $\boldsymbol{r_{GPS}}$. A similar 'lever-arm' vector from the INS to the sensor $\mathbf{r_{INS\text{-}SEN}}$ relates platform position, attitude and velocity information to the sensor's *Record Reference System* located at the lens projection center.

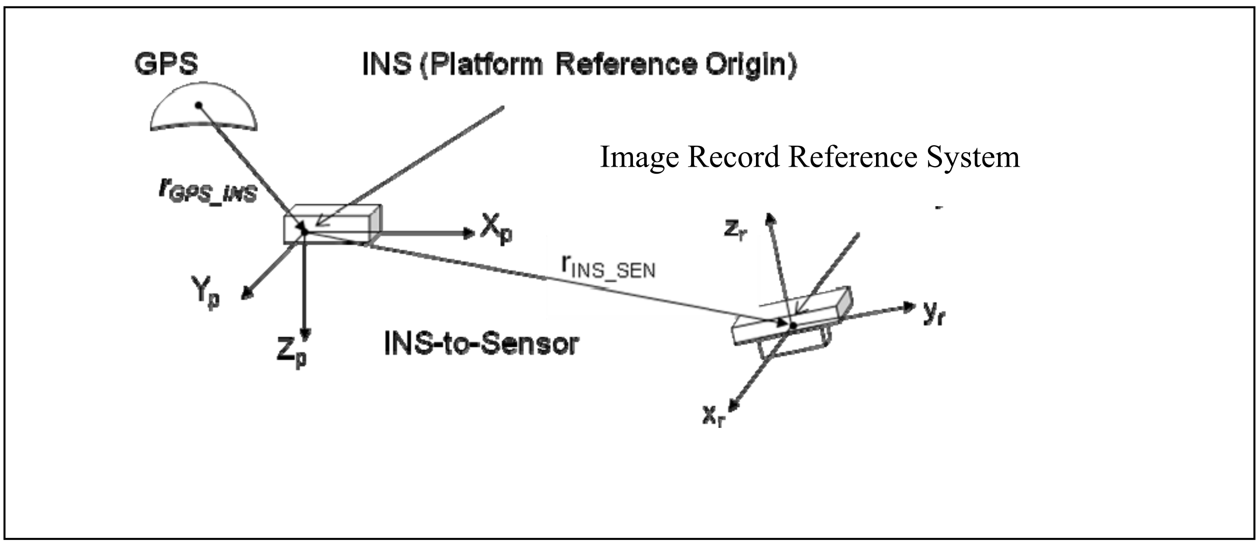

Figure 1. Nominal Relative GPS to INS to Sensor Relationship

2.1.1 Nomenclature

The terms Inertial Measurement Unit (IMU) and Inertial Navigation System (INS) are often confused. An IMU is an instrument that measures specific forces and angular rates relative to an inertial frame of reference. An INS contains an IMU as one of its components, but also includes the ability to use the IMU measurements to derive meaningful position, velocity and attitude information.

The IMU actually measures specific forces, which are related to the applied accelerations through the gravity field. The Inertial Navigation System (INS), which contains an IMU as one of its components, integrates the rotation rates to obtain orientation changes, iteratively integrates the accelerations to first obtain velocities and doubly integrates the accelerations to obtain position increments (Jekeli, 2000). It employs the Kalman filtering mathematical process that estimates the correct state of a system from measurements that contain random errors. The integration of the rotation rates implies that vehicle orientation is obtained as a natural byproduct of the navigation solution, thus adding potentially useful information to certain applications since orientation is not usually a product of GPS-only systems. The following block diagram (Figure 2) illustrates the typical GPS/INS process.

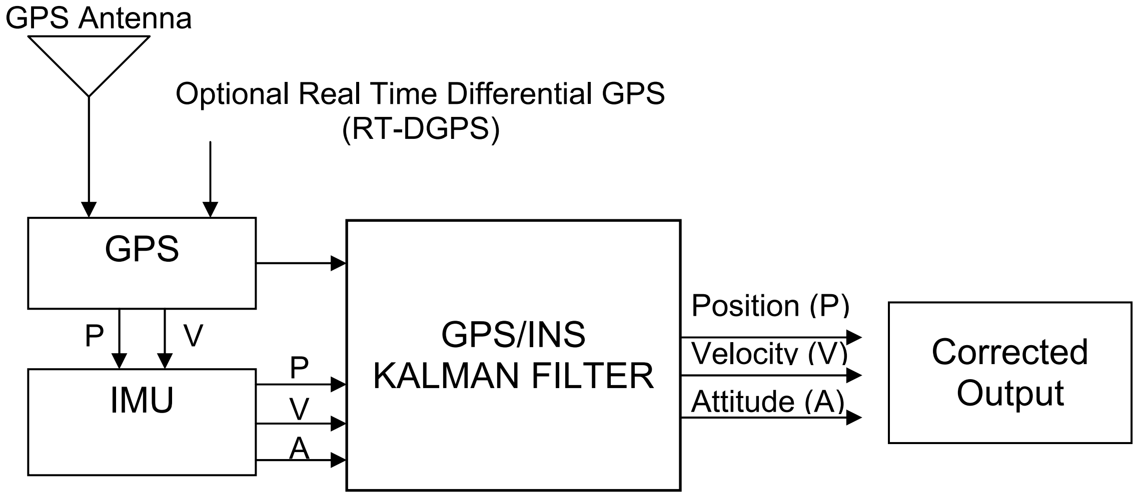

Figure 2. GPS / INS Processing Block Diagram

Furthermore, the integration process acts as a low-pass filter and thus produces very accurate short-term position and velocity differences. Also, in contrast to GPS which typically updates position and velocity at 1 to 20 Hz, the IMU is capable of making measurements at several hundred Hz. Although rarely processed at this rate, IMU output rates of 50 Hz or higher are not uncommon. Despite the above advantages, sensor inaccuracies such as gyro drifts and accelerometer biases cause a rapid degradation in pure-inertial position quality. To this end, higher quality IMUs, obtained at significantly higher cost, exhibit significantly slower position degradation. However, in many applications, aviation and satellites being obvious ones, the traditional approach to obtain Zero Velocity Updates (ZUPTs) through periodic stops of the vehicle are impractical, if not impossible. Such applications therefore require either a very accurate IMU or another means of bounding the errors. Given the complimentary nature of GPS and INS, their integration represents the best opportunity for meeting the ever-increasing accuracy and availability demands of commercial users. The advantages of GPS/INS integrated systems, relative to GPS or INS only, are reported to be a full position, velocity and attitude solution, improved accuracy and availability, smoother trajectories, greater integrity and reduced susceptibility to jamming and interference. The inertial solution also enhances GPS ambiguity resolution performance. These benefits have been exploited for a wide variety of applications including airborne mapping, airborne positioning, and mobile mapping systems.

Position accuracy during complete GPS data outages (i.e., absence of updates) is a direct reflection of system performance. An INS is the perfect complement to GPS. Orthogonally-mounted accelerometers and angular rate sensors (gyros) that comprise the IMU which when combined with the mechanization equations (and system error estimation) defines the INS itself. The identified offsets in position and attitude (GPS to INS to sensor) must be incorporated in the collinearity equations described in Section 4.

2.2 Frame Sensor

The purpose of a physical frame sensor model is to develop the mathematical relationship between the position of an object on the Earth's surface and its image as recorded by an overhead sensor. Historically frame film cameras or digital sensors were single nadir viewing devices without embedded GPS and IMU functionality. The continued development of digital sensors since 2004 has extended the designs to incorporate systematic oblique photography for mapping reconnaissance and visualization purposes. They include fan, block and five camera "Maltese Cross" configurations from system houses such as Track'Air, Rolleimetric, IGI, and DIMAC Systems. A more unique implementation from VisionMap, Ltd. provides a stepping frame camera that produces a systematic series of digital oblique photographs in the cross-track direction for wide angular coverage. One design example is the Arched Retinal Camera Array (ARCA) of the Integrated Retinal Imaging System (IRIS) from M7 Visual Intelligence LP (Figure 3).

The consistent fact for all these current designs is that they employ sets of individual cameras which can be calibrated and exploited using the mathematical development of this document and nearly all incorporate embedded GPS and IMU functionality.

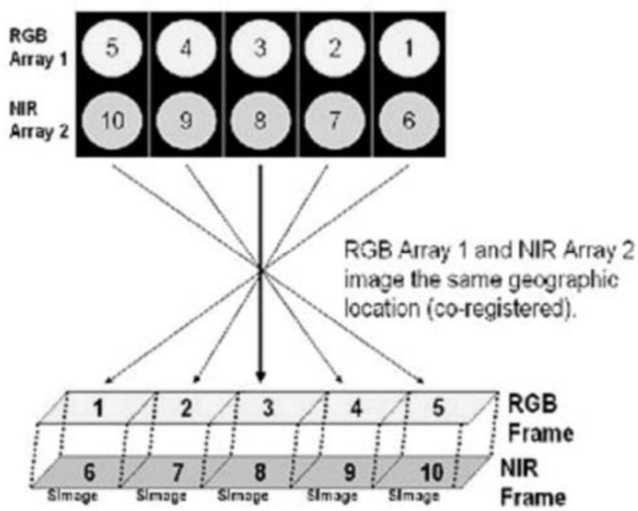

Figure 3. Arched Retinal Camera Array

Therefore, the optimal set parameters required to relate the image record to the object remains to be the *interior orientation* parameters of the sensor (focal length, principal point offset, radial and tangential distortion) and the six *exterior orientation* parameters of position and orientation for the sensor at each exposure time. Also, if the interior orientation items are precisely known from a sensor calibration document, and error estimates are available for each interior and exterior orientation component, then the process of computing an accurate geographic position for imaged objects can be directly implemented in the collinearity equations. If the complete set of parameters and error estimates are not known and must be developed from other platform and sensor data elements.

An image's spatial position may be given, at least initially or in its raw form, either in relation to a coordinate system locally defined or relative to an Earth reference. A horizontal (latitude and longitude) and a vertical (elevation) datum will be required to define the origin and orientation of the coordinate systems. Likewise, the corresponding object's position may be defined with respect to either that same coordinate system, or attached to any number of Earth-based datum. For purposes of this metadata profile, transformation between the various coordinate systems will be accomplished via a sequence of translations and rotations of the sensor's coordinate system origin and axes until it coincides with an Earth-based coordinate system origin and axes. An overall view of some of the coordinate system reference frames under consideration is shown in Figure 4.

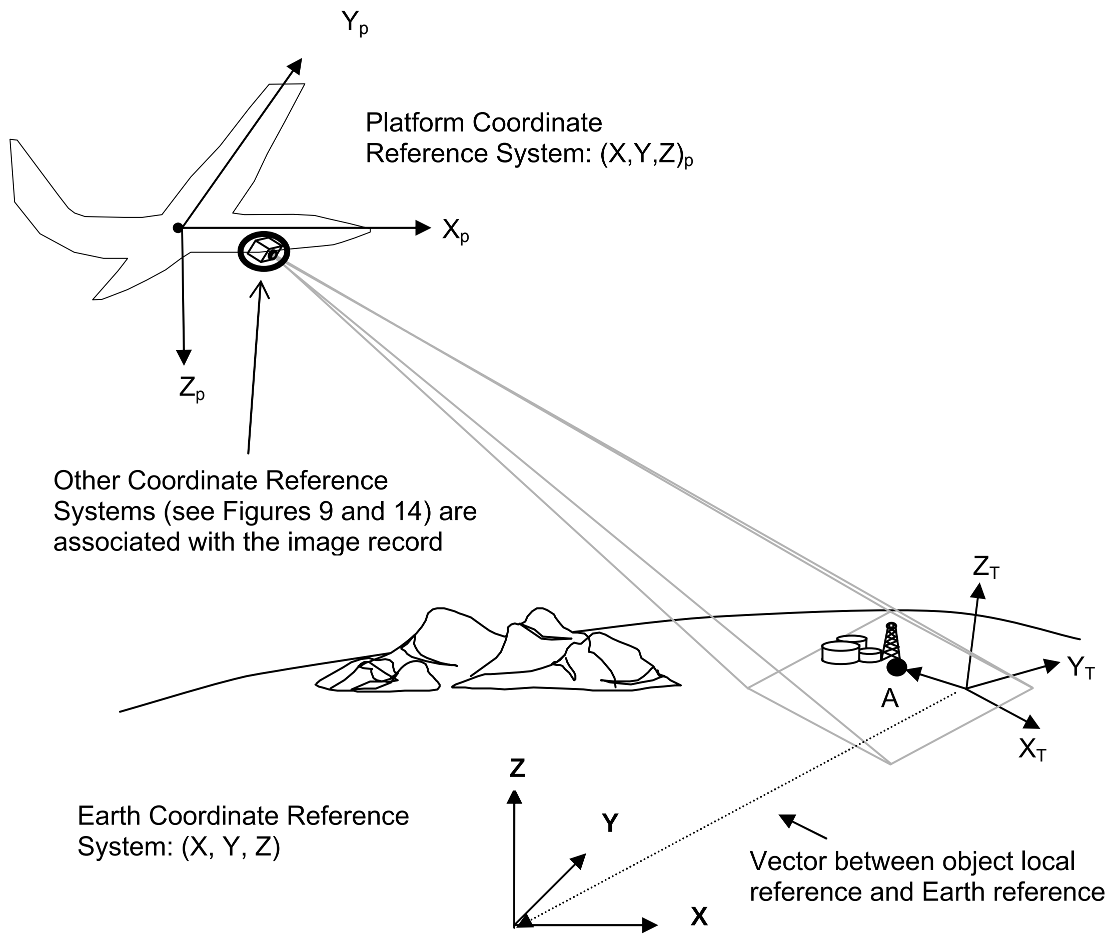

Figure 4. Multiple coordinate reference frames

2.3 Earth Coordinate Reference System

To simplify the frame sensor model development, a stationary, non-time dependent coordinate reference frame is needed to which all other reference frames may be mathematically defined. An Earth-Centered, Earth-Fixed (ECEF) coordinate system (X,Y,Z) as shown in Figure 5 was selected. The ECEF system is defined such that the X-Y plane is parallel to the equator, the **X-axis** intersects the Greenwich Meridian (from where longitude is measured; longitude equals 0-degrees at X equal to zero), the **Z-axis** is parallel to the Earth's rotation axis and points toward the North pole, the **Y-axis** is in the equatorial plane and perpendicular to **X** and completes a right-handed coordinate system, i.e., the cross-product of **X** and **Y** is a vector in the direction of **Z**.

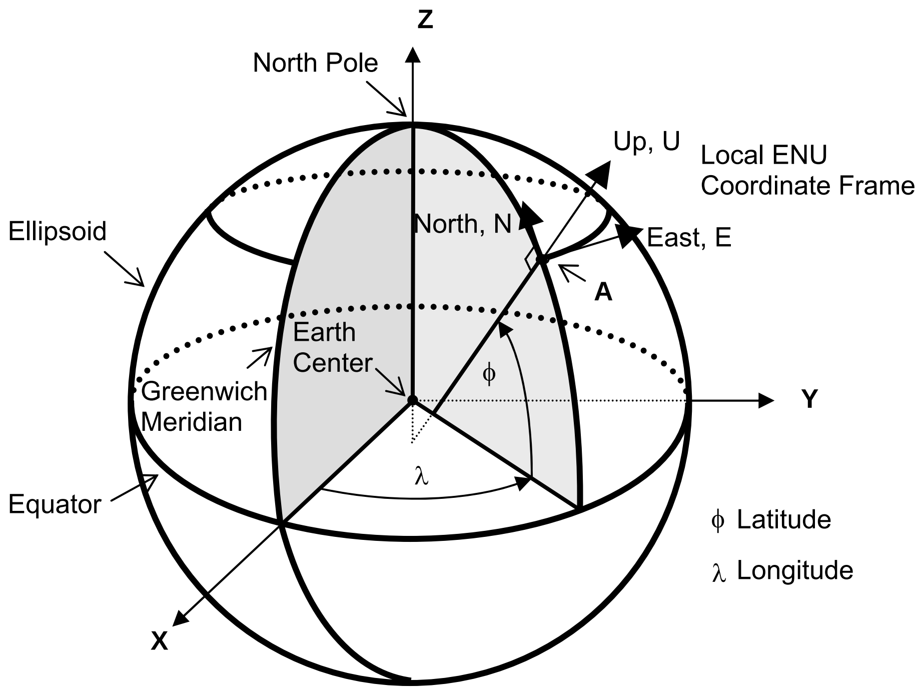

Figure 5. Earth-centered and local surface (ENU) coordinate frames (MIL-STD-2500C)

Therefore, any point (A) on the reference surface may be described in (X,Y,Z) coordinates, or alternatively in the equivalent longitude, latitude, and elevation terms. Likewise, this point, the “object” point, can be described relative to a local reference system attached to the surface, specifically in an East-North-Up (ENU) orientation; where the North vector is tangent to the local prime meridian and pointing North, the Up vector points to the local zenith, and East vector completes a right-hand Cartesian coordinate system.

2.4 Platform Coordinate Reference System

The platform coordinate reference system is defined with respect to its center of navigation, fixed to the platform structure, e.g., the aircraft as shown in Figure 6. The axes are defined as: Xp positive along the heading of the platform, which is the platform roll axis; Yp positive in the direction of the starboard (right) wing, along the pitch axis such that the XpYp plane is horizontal when the aircraft is at rest; and Zp positive down, along the yaw axis. A second platform coordinate reference system is also defined with respect to a North-East-Down (NED) reference system with its origin at the center of navigation. In horizontal flight the platform Zp axis is aligned with the Down (D) axis, and the North-East plane is parallel to the tangent plane to the Earth surface reference ellipsoid at the intersection of the D axis, Figure 7. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as pitch, roll and yaw that are a specific sequence of Euler angles used in aerospace applications to define the relative orientation of the vehicle. The three angles specified in this formulation are defined as the roll angle, pitch angle and yaw angle.

Therefore, the platform reference system orientation defined in terms of its physical relation (rotation) about this local NED reference, Figure 6, are as follows:

Platform heading - angle from the north axis of the NED, measured in the horizontal plane, to the projection of the platform positive roll axis, $X_{p,}$, in the horizontal plane (positive from north to east).

Platform pitch - angle from the NED horizontal plane to the platform positive roll axis, X_p-axis (positive when $+X_p$ is above the NED horizontal plane, or nose up).

Platform roll - rotation angle about the platform roll axis; positive if the platform positive pitch axis, Y_p, lies below the NED horizontal plane (right wing down).

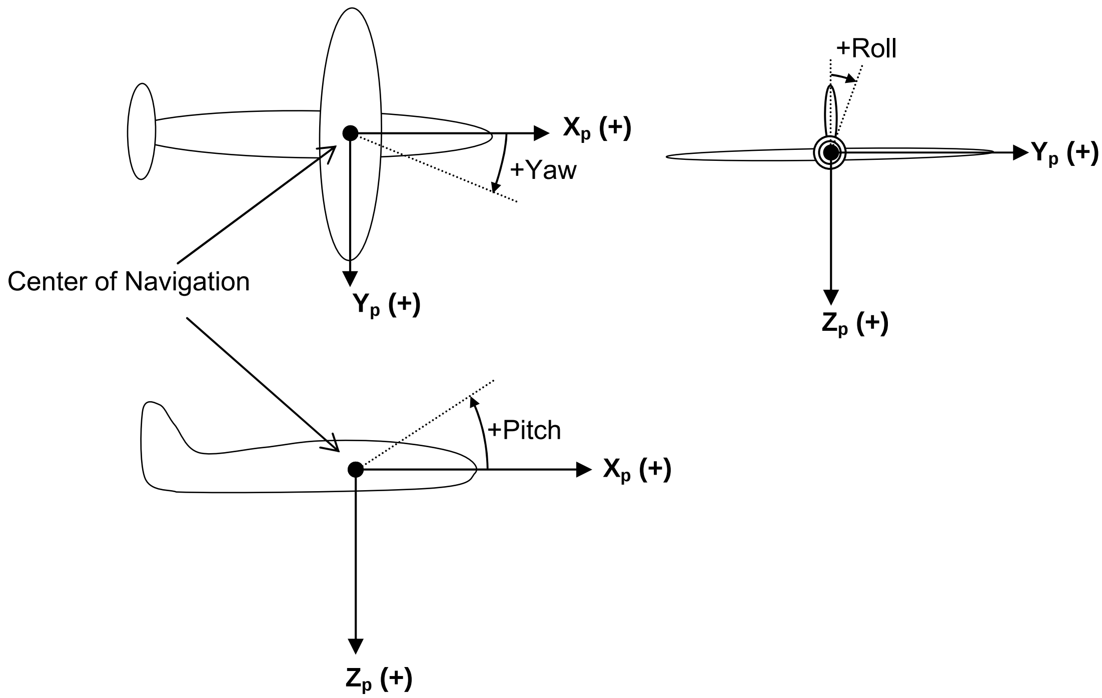

Figure 6. Platform coordinate reference system and local (NED) frame

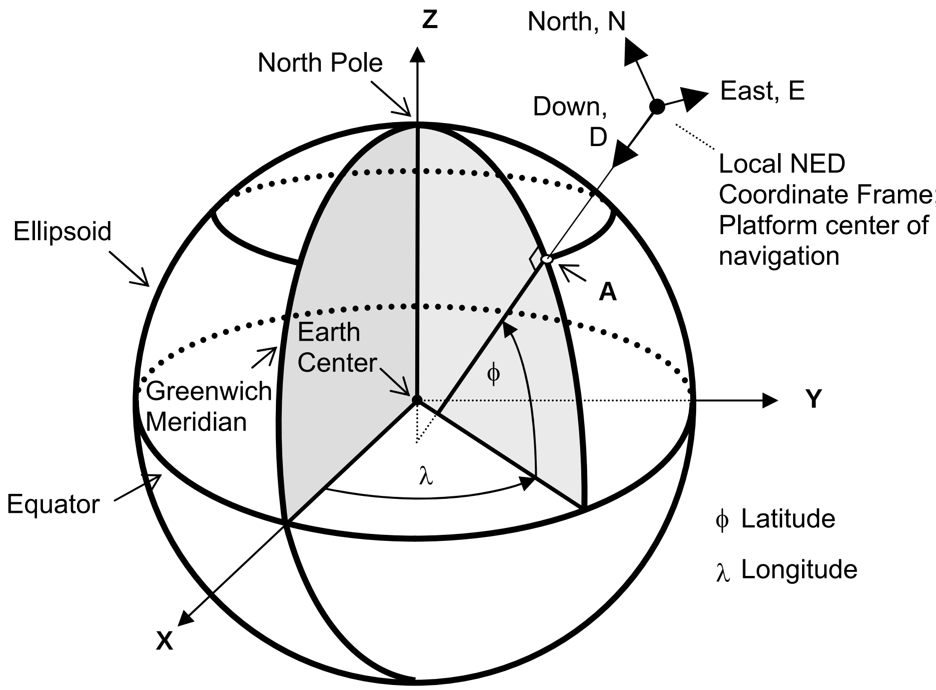

Figure 7. Earth and local platform (NED) coordinate frames

The NED can be further defined to relate the local platform center of navigation through a sequence of angular rotations to the local Earth surface (ENU) reference; that is, the latitude, longitude and height, relative to an Earth-based ellipsoidal datum (e.g., WGS-84, Tokyo, etc.) and a vertical reference such as Mean Sea Level (MSL) or Earth Gravity Model 1998 (EGM-98). In turn, this local surface-based ENU reference can be translated and rotated into the ECEF frame; that is, latitude, longitude and gravity vector-based reference such as WGS-84, latitude (positive north), longitude (positive east) and gravity vectors relative to the Earth-based datum (i.e., WGS-84, EGM-98 ellipsoid).

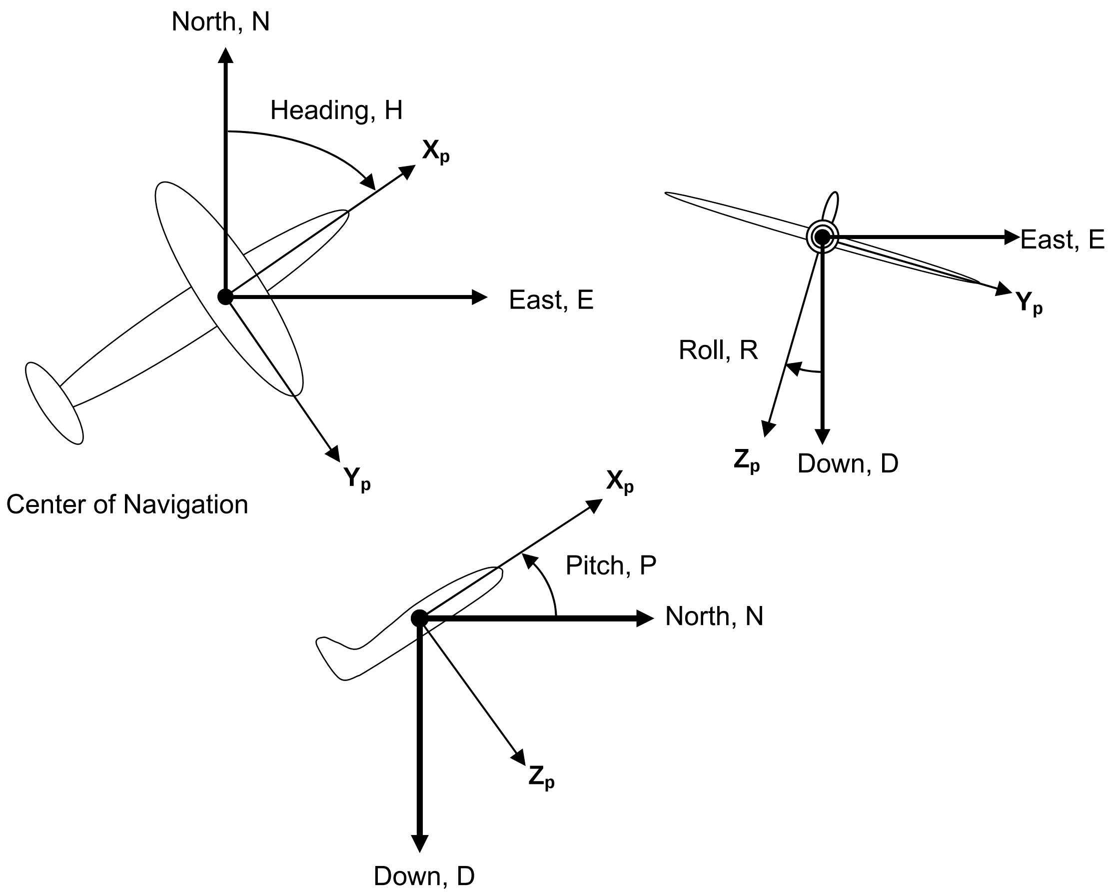

Figure 8. Platform coordinate reference system and local (NED) frame

3. Frame Sensor Interior Descriptions

3.1 Sensor Coordinate Reference System

The origin of the sensor coordinate system is defined to be centered on the frame sensor lens at its "lens center" (called *nodal point*, in optics) or "perspective center" (xr,yr,zr) (Figure 10).

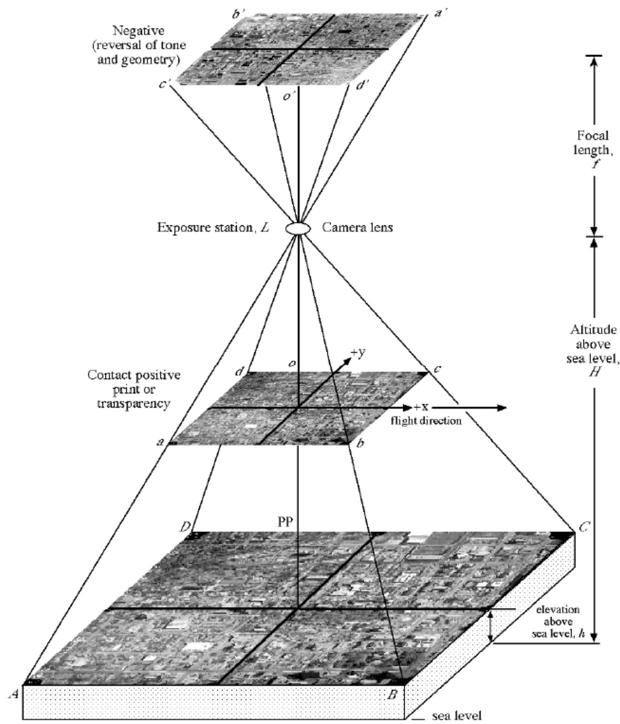

Figure 9. Frame Camera / Sensor Example

All photogrammetric development in this document is based on the use of the positive image located between the exposure station (or perspective center) and the ground. The image record reference coordinate system is defined (Figure 9 and Figure 10), such that the positive *x*-axis is in the direction of an image row (increasing column indices) and the *z*-axis is along the optical axis which is perpendicular to the lens plane, and pointing away from the collection array. To establish an unambiguous alignment between the image record and platform reference systems, at rest, the *z*-axis (z_r) will be parallel, but in opposite direction, to the platform Z_p–axis at nadir.

Depending on the sensor physical installation, the sensor coordinate system may be reported directly to the gimbals to which the sensor is attached or relative to the platform's center of navigation (INS), which in turn may be referenced to the GPS or other datum based coordinate reference system as described in Section 2.1. Since gimbal information is unique to each sensor/platform design, the intermediate rotations and translations required to align these specific components will not be addressed. However, an example case is treated in Appendix A.

For collectors that are cameras, i.e., film-based, factors that may not pertain to a digital sensor must be accounted for, e.g., distortion factors associated with film deformation. These film distortions are accounted for in Section 3.3. Although digital sensors may not suffer exactly these same distortion factors, they may have their own unique distortions, such as unevenly spaced elements, which can be treated by the same equations. The transformation from line/sample to x,y coordinates will accommodate both media, as will also be shown in Section 3.3.

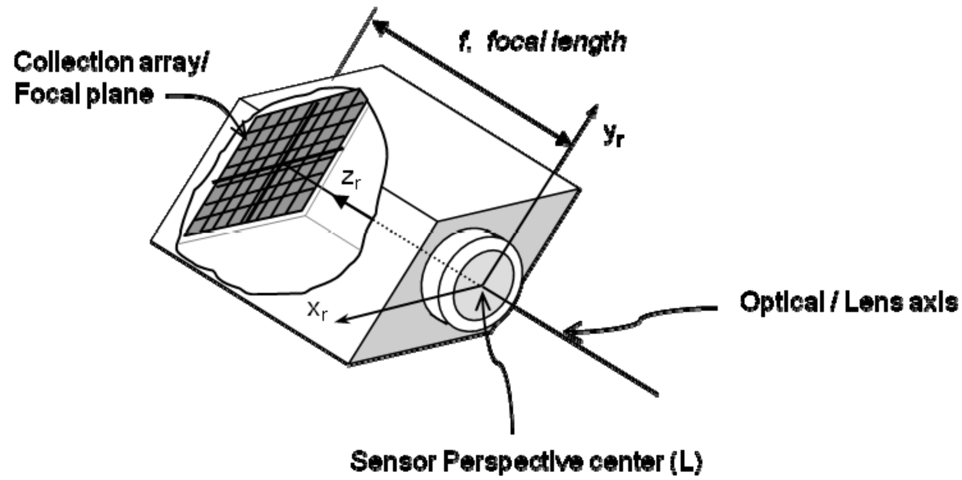

Figure 10. Image Record reference frame

3.2 Typical Imagery Sensor Storage Layout

Typical of common imagery formats, and in particular ISO/IEC 12087-5, picture elements (pixels) are indexed according to placement within a “Common Coordinate System” (CCS), a two-dimensional array of rows and columns, as illustrated in the array examples in Figure 11. There are three commonly referred to coordinate systems associated with digital and digitized imagery: row, column (r,c), line, sample (ℓ,s), and x,y. The units used in the first two systems are pixels (and decimals thereof), while the x,y are linear measures such as mm (and decimals thereof), as will be introduced in following subclauses. The origin of the CCS, as shown in Figure 11, is the upper left corner of the first (or 0,0) pixel, which in turn is the upper left of the array. Because the CCS origin is the pixel corner, and the r,c associated with a designated pixel refers to its center, the coordinates of the various pixels, (0.4,0.5), (0.5,1.5), … etc., are as shown in Figure 11.

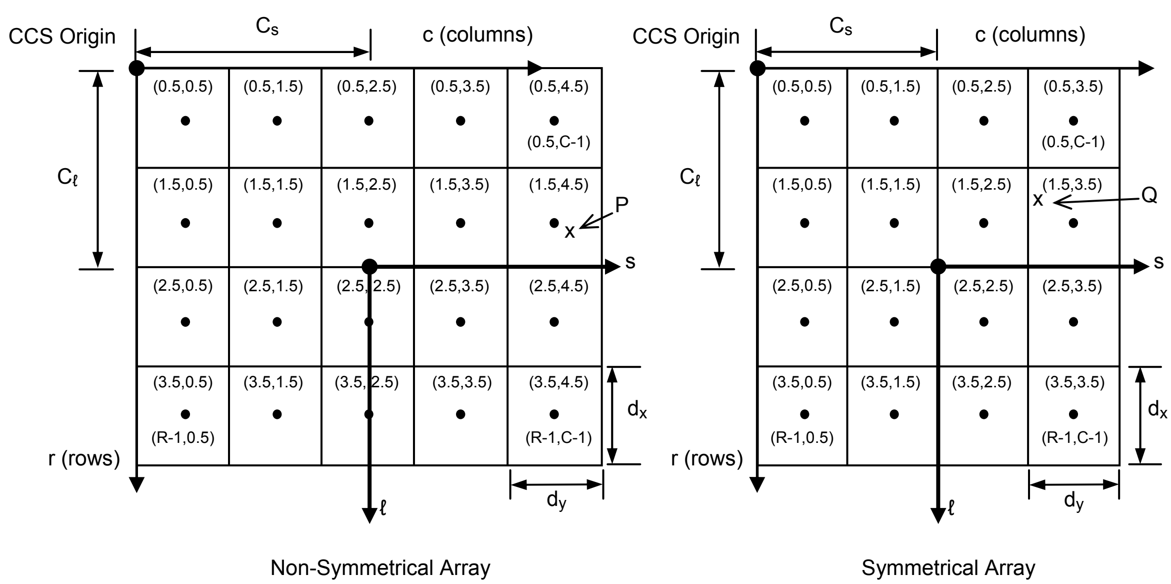

Figure 11. Pixel orientation within the frame sensor coordinate system

3.3 Row, Column (r,c) to Line, Sample (ℓ,s) Coordinate Transformation

Since all mathematical development to follow are based on geometric center of the image as the origin, the (r,c) system is replaced by the (ℓ,s) system through two simple translations:

$$\ell = r - C_\ell$$
$$s = c - C_s$$
Eq. 1

where C_ℓ and C_s are each half the image pixel array size, in pixels, in the row and column directions, respectively.

Examples.

Figure 11(Nonsymmetrical Array): For $C_\ell = 4/2 = 2.0$ $C_s = 5/2 = 2.5$ $r_p = 1.6\ pixel$

$$c_p = 4.7\ pixel$$
$$\ell_p = r_p - C_\ell = 1.6 - 2.0 = -0.4\ pixel$$
$$s_p = C_p - C_s = 4.7 - 2.5 = 2.2\ pixel$$

Figure 11 (Symmetrical Array): For $C_\ell = 4/2 = 2.0$ C_s, $= 4/2 = 2.0$ $r_Q = 1.4\ pixel$

$$c_Q = 3.1\ pixel$$
$$\ell_Q = r_Q - C_\ell = 1.4 - 2.0 = 0.6\ pixel$$
$$s_Q = c_Q - C_s = 3.1 - 2.0 = 1.1\ pixel$$

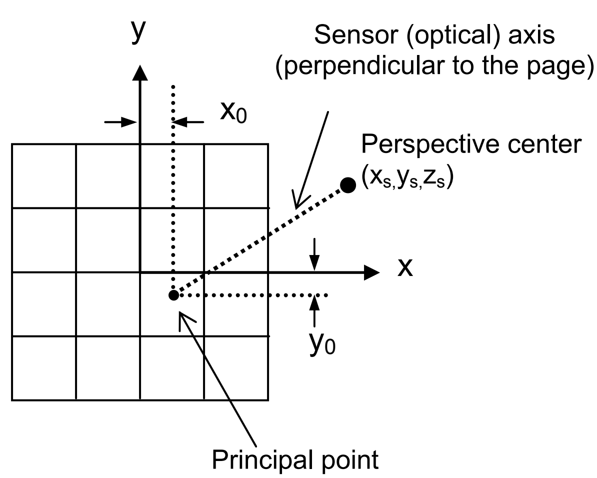

Figure 12. Placement of sensor axis

Interior distortions or flaws, e.g., lens distortion errors, must be applied before the pixel-to-image transformation. Sensor developers may account for these imperfections as a part of calibration or through the use of other testing techniques to assess their sensor performance. In those cases, no adjustment by the exploitation tool may be necessary if lookup tables or automatic corrections are provided. Note that this paper will provide a simplified development and not attempt to model all of the possible influences, such as sensor array warping due to temperature changes, timing or dwell of image capture. Should applications require such advanced considerations, those influences, properly modeled, could be inserted into this basic model.

3.3.1. Array and Film distortions

Deformations in the imagery are accounted for as follows:

$$\ell = a_0 x + a_1 y + a_2$$
$$s = a_3 x + a_4 y + a_5$$

Eq. 2 Error! Bookmark not defined.

This transformation accounts for two scales, a rotation, skew, and two translations. The six parameters are usually estimated on the basis of (calibrated) reference points, such as camera fiducial marks, or their equivalent corner pixels for digital arrays. Here, the (x,y) image coordinate system, as shown in Eq. 2, is used in the construction of the mathematical model, and applies to both film and digital sensors.

3.3.2. Principal point

Ideally the sensor (lens) axis would, as in Figure 10, intersect the collection array at its center, (x,y) or (0,0). However, this is not always the case due to lens flaws, imperfections, or design, and is accounted for by offsets x0 and y0, as shown in the figure. Note that x0 and y0 are in the same linear measure (e.g., mm) as the image coordinates (x,y) and the focal length, *f*. For most practical situations, the offsets are very small, and as such there will be no attempt made to account for any covariance considerations for these offset terms.

3.3.3. Optical distortions

Effects due to optical (lens) distortion are measured in terms of **radial** components. Assuming that calibration factors are not provided, the radial distortion can be approximated by a polynomial function applied to the x and y components, see Figure 13. The polynomial may take different forms, e.g., odd powers of the radial distance, or a scalar applied to the square of the radial distance. For purposes of this development, we follow a modified NGA Generic Sensor Model algorithm as follows:

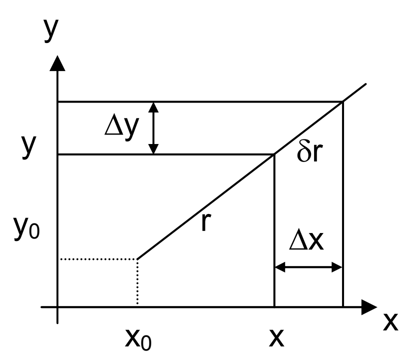

Figure 13. Radial optical distortion

$$\delta r = k_1 r^3 + k_2 r^5 + k_3 r^7$$ **Eq. 3**Error! Bookmark not defined.

where:

$$r = \sqrt{\bar{x}^2 + \bar{y}^2} \qquad \bar{x} = x - x_0 \qquad \bar{y} = y - y_0$$ **Eq. 4**Error! Bookmark not defined.

and k represents the third-, fifth-, and seventh-order radial distortion coefficients. In most practical situations, the influence of the seventh-order term (k_3) is insignificant and can be ignored. This term will be included within the equations below, but not carried forward in the derivation of the collinearity equations. These k coefficients are obtained by fitting a polynomial to the distortion curve data either from camera calibration data or the least squares adjustment output of the collinearity equations extended for these data. Contributions of first-order terms may also be accomplished via adjustment to the focal length, but we have chosen to maintain attribution of distortion effects with their associated polynomials. The influence of this distortion is typically described as either a “pincushion” or “barrel” distortion, as shown in Figure 14.

The resultant x and y radial optical distortion components are then:

$$\Delta x_{radial} = \bar{x}\frac{\delta r}{r} = \bar{x}\frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r} = \bar{x}(k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$\Delta y_{radial} = \bar{y}\frac{\delta r}{r} = \bar{y}\frac{k_1 r^3 + k_2 r^5 + k_3 r^7}{r} = \bar{y}(k_1 r^2 + k_2 r^4 + k_3 r^6)$$ **Eq. 5**Error! Bookmark not defined.

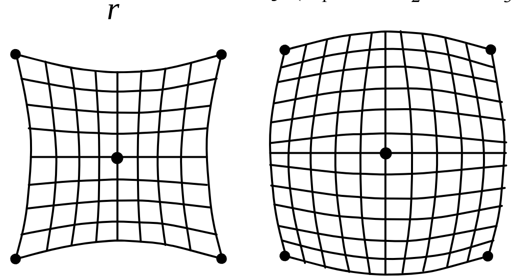

Figure 14. Optical radial distortion effects

Another interior imperfection is described in terms of rotational symmetry, or “**decentering**.” While, in general, these effects may be assumed to be minimal, they may be more prominent in variable focus or zoom cameras. Consideration of this effect is given via the following (NIMA System Generic Model):

$$\Delta x_{decen} = p_1(2\bar{x}^2 + r^2) + p_2(2\overline{xy})$$
$$\Delta y_{decen} = p_1(2\overline{xy}) + p_2(2\bar{y}^2 + r^2)$$ **Eq. 6**Error! Bookmark not defined.

where Δx_{decen} and Δy_{decen} are the x and y components of the decentering effect, respectively; p1 and p2 are decentering coefficients. Note that the referenced document included a third coefficient; however, for all practical purposes, this term’s influence is so small that we choose to ignore it. Therefore, the contributions of lens radial distortions and decentering of x and y components are:

$$\Delta x_{lens} = \Delta x_{radial} + \Delta x_{decen} = \bar{x}(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1(2\bar{x}^2 + r^2) + p_2(2\overline{xy})$$
$$\Delta y_{lens} = \Delta y_{radial} + \Delta y_{decen} = \bar{y}(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1(2\overline{xy}) + p_2(2\bar{y}^2 + r^2)$$

Eq. 7

3.3.4. Atmospheric Refraction

Adjustments may be required to account for bending of the image ray path as a result of atmospheric effects. These influences generally increase as altitude and look angles increase. Several methods of varying complexity are available to approximate the needed adjustments, including, for example, consideration of temperature, pressure, relative humidity and wavelength. For purposes of this paper, we have chosen to adopt the following simple approximation (Mikhail, 2001), where α is the angle the refracted ray makes with the local vertical, the angular displacement Δd (micro-radians) then becomes:

$$\Delta d = K \tan\alpha$$

where

$$K = \frac{2410 \times H_{msl}}{H_{msl}^2 - 6H_{msl} + 250} - \frac{2410 \times h_{msl}}{h_{msl}^2 - 6h_{msl} + 250}\left(\frac{h_{msl}}{H_{msl}}\right)$$

and

$$\alpha = \tan^{-1}(r/f),$$

Eq. 8

H_{msl} is altitude (km, MSL) of the sensor,
h_{msl} is the object elevation (km, MSL),
and K is the refraction constant (micro-radians).

This equation is a good approximation for collection parameters resulting when the optical axis coincides with the vertical axis (**Z**T,) from the ground object. Depending on the level of precision required, off-vertical collections may require more rigorous models. This development of a "standard" sensor model proposes to use units of meters, referenced to height above ellipsoid (HAE), for sensor altitude and object elevation, thus the distinction between H_{msl}, h_{msl} (km, MSL) as used in the above equation and H, h (m, HAE) in the forthcoming standard is highlighted here.

Therefore, given image coordinates ($\bar{x}$, $\bar{y}$), the resulting coordinates (x'_{ref},y'_{ref}) are:

$$x'_{ref} = \bar{x}\frac{r'_{ref}}{r}$$
$$y'_{ref} = \bar{y}\frac{r'_{ref}}{r}$$

where

$$r = \sqrt{\bar{x}^2 + \bar{y}^2}$$

and

$$r'_{ref} = f\tan(\alpha + \Delta d)$$

Eq. 9

It follows, then, that the refraction correction components (Δx_{ref}, Δy_{ref}) are:

$$\Delta x_{ref} = x'_{ref} - \bar{x} = \bar{x}\left(\frac{r'_{ref}}{r} - 1\right)$$
$$\Delta y_{ref} = y'_{ref} - \bar{y} = \bar{y}\left(\frac{r'_{ref}}{r} - 1\right)$$

Eq. 10

Lastly, the corrections to the original image coordinates (*x,y*) are combined to establish the corrected image coordinates as follows:

$$x' = \bar{x} + \Delta x_{lens} + \Delta x_{ref}$$
$$= \bar{x} + \bar{x}(\mathrm{k}_1 r^2 + \mathrm{k}_2 r^4 + \mathrm{k}_3 r^6) + \mathrm{p}_1(2\bar{x}^2 + r^2) + \mathrm{p}_2(2\overline{xy}) + \bar{x}\left(\frac{r'_{ref}}{r} - 1\right)$$
$$y' = \bar{y} + \Delta y_{lens} + \Delta y_{ref}$$
$$= \bar{y} + \bar{y}(\mathrm{k}_1 r^2 + \mathrm{k}_2 r^4 + \mathrm{k}_3 r^6) + \mathrm{p}_1(2\overline{xy}) + \mathrm{p}_2(2\bar{y}^2 + r^2) + \bar{y}\left(\frac{r'_{ref}}{r} - 1\right)$$

Eq. 11

where *x'* and *y'* are the resulting corrected image coordinates.

Simplifying Equation 11:

$$x' = \bar{x}(\mathrm{k}_1 r^2 + \mathrm{k}_2 r^4 + \mathrm{k}_3 r^6 + \frac{r'_{ref}}{r}) + \mathrm{p}_1(2\bar{x}^2 + r^2) + \mathrm{p}_2(2\overline{xy})$$
$$y' = \bar{y}(\mathrm{k}_1 r^2 + \mathrm{k}_2 r^4 + \mathrm{k}_3 r^6 + \frac{r'_{ref}}{r}) + \mathrm{p}_1(2\overline{xy}) + \mathrm{p}_2(2\bar{y}^2 + r^2)$$

Eq. 12

Therefore, given pixel coordinates (r,c), calculating the image coordinates, including correction factors considered, may be accomplished through the use of Equations 1, 2, 4, 8, 9, and 12. Therefore, (*x',y'*) are the coordinates required to establish the image-to-object transformation.

3.4 External Influences

3.4.1. Curvature of the Earth

Adjustments are required to account for curvature of the Earth if transforming between rectangular (Cartesian) coordinates to earth coordinates given in map projection (e.g., Lambert Conformal or Transverse Mercator). There are software programs available which provide the transformation between these differing formats, e.g., GEOTRANS; these will not be included in this development. Instead, we shall maintain use of Cartesian coordinates throughout.

This mathematical development applies to well-calibrated metric cameras/sensors where all known systematic errors/distortions are corrected for before applying the fundamental imaging equations for central perspective applicable to frame imagery as introduced in the next section.

4. Collinearity Equations

4.1 Development based on North-East-Down Coordinate Reference System

The derivation of the relationship between image coordinates and the corresponding object coordinates on the Earth's surface requires the development of the relationship between the image and the object coordinate systems. This process is accomplished via translation, rotation and scaling from one coordinate system to the other.

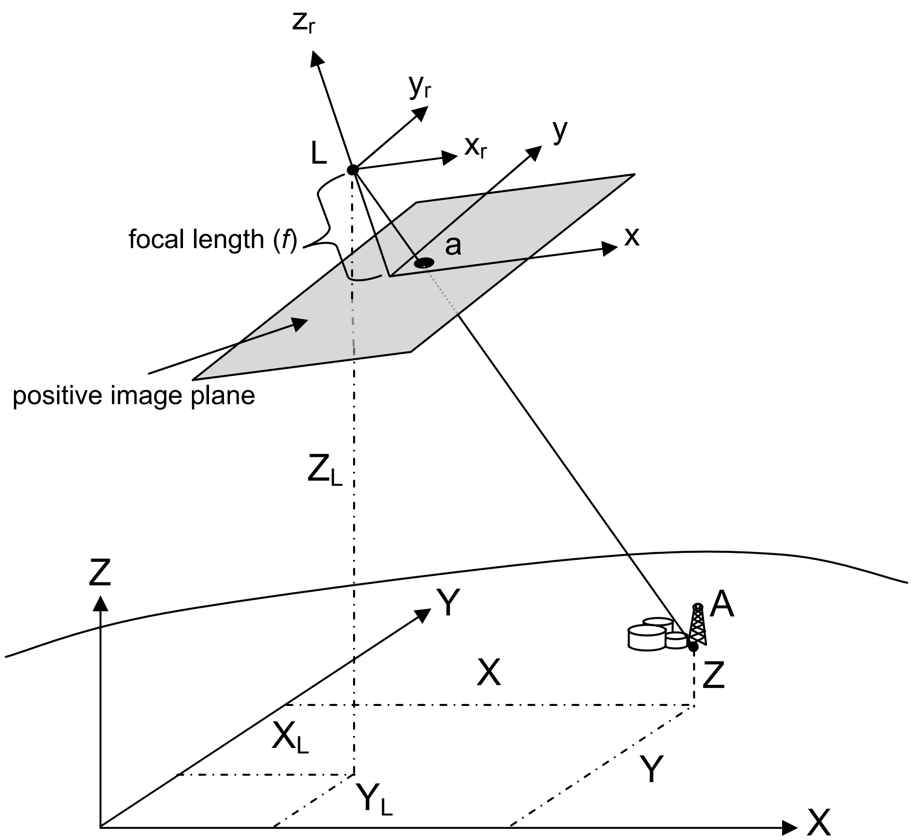

Figure 15. Collinearity of sensor perspective center, image, and corresponding object point

Geometrically, the collinearity condition enforces the fact that the sensor perspective center (L), the "ideal" image point (**a**), and the corresponding object point (**A**) are collinear. Note that the "ideal" image point is represented by image coordinates *after* having been corrected for all systematic effects (array or film deformations, lens distortions, atmospheric refraction, etc.), as given in the preceding section.

For two vectors to be collinear, one must be a scalar multiple of the other. Therefore, vectors from the perspective center (L) to the image point and object point, **a** and **A** respectively, are directly proportional. Further, in order to associate their components, these vector components must be defined with respect to the same coordinate system. Therefore, we define this association via the following equation:

$$\mathbf{a} = k\mathbf{M}\mathbf{A} \quad \textbf{Eq. 13}$$

where k is a scalar multiplier and **M** is the orientation matrix that accounts for the rotations (roll, pitch and yaw) required to place the Earth coordinate system parallel to the sensor's image reference coordinate system. Therefore, the collinearity conditions represented in Figure 15 become:

$$\left[\begin{pmatrix} x_a \\ y_a \\ 0 \end{pmatrix} - \begin{pmatrix} x_o \\ y_o \\ f \end{pmatrix}\right] = \mathbf{kM}\left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_L\right] \qquad \textbf{Eq. 14}$$

where:

x_a, y_a are image coordinates of a point,
x_o, y_o are the principal point coordinates,
f is the focal length,
k is a scale factor,
X_L, Y_L, and Z_L are the coordinates of the lens perspective centre, L, in the world coordinate system; and
X, Y, and Z are the coordinates of the object point, A, in the world coordinate system.

The orientation matrix **M** is the result of three sequence-dependent rotations:

$$\mathbf{M} = \mathbf{M}_\kappa \mathbf{M}_\phi \mathbf{M}_\omega = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \qquad \textbf{Eq. 15}$$

The rotation ω is about the X-axis (roll), ϕ is about the once rotated Y-axis (pitch), and κ is about the twice rotated Z-axis (yaw). Multiplying out the three matrices in Eq. 15, the orientation matrix **M** becomes:

$$\mathbf{M} = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix} \qquad \textbf{Eq. 16}$$

Referring to Figure 8, all development has been with respect to the positive image plane which is also shown in Figure 15. The image point a in Equation 14 is represented by coordinates (xa, ya).

Therefore, for any given object, its "World" coordinates (X,Y,Z) are related to the coordinates (x_a,y_a) by the following pair of collinearity equations that result from the manipulation of Eq. 14 to eliminate the scalar, k:

$$x_a = x_0 - f\left[\frac{\cos\phi\cos\kappa(X - X_L) + (\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa)(Y - Y_L) + (\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa)(Z - Z_L)}{\sin\phi(X - X_L) - \sin\omega\cos\phi(Y - Y_L) + \cos\omega\cos\phi(Z - Z_L)}\right]$$
$$y_a = y_0 - f\left[\frac{\cos\phi\sin\kappa(X - X_L) + (\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa)(Y - Y_L) + (\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa)(Z - Z_L)}{\sin\phi(X - X_L) - \sin\omega\cos\phi(Y - Y_L) + \cos\omega\cos\phi(Z - Z_L)}\right] \qquad \textbf{Eq. 17}$$

The coordinates xa and ya above represent "corrected" pair, (*x'*,*y'*), from Equation 12. These equations above also rely upon the positional and orientation information of the sensor. Unfortunately, the ability to accurately measure the sensor position, e.g., system latency, GPS/INS errors, can be the source of a substantial amount of uncertainty. The recent shift in interest from simply providing an "image" and the visual information it encompasses, to exploitation of the image to provide highly accurate coordinates serves to highlight this difficult challenge. The degree to which the accuracy of these results is required will determine the degree to which modeling of the collection system parameters is required.

From the elements of **M**, the three angles may be calculated:

$$\omega = \arctan\left(-m_{32}/m_{33}\right),$$

$$\varphi = \arctan\left(m_{31}\right), \text{ and}$$

$$\kappa = \arctan\left(-m_{21}/m_{11}\right).$$

For the arctan function, the signs of both the numerator and denominator of the argument must be used to select the correct quadrant.

5. Application of Sensor Model

5.1 Adjustable Parameters

The development in the preceding Sections addresses all sensor and platform interior and exterior parametric information associated with the creation of a physical frame sensor model.

In practice, if the interior orientation parameters of the sensor (focal length, principal point offset, radial and tangential distortion) are precisely known from a calibration document, if the six exterior orientation parameters for the sensor at each exposure time are also precisely known, and if error estimates are available for each component, then the process of computing an accurate geographic position for imaged objects can be directly implemented in the collinearity equations.

Since this information is normally not available directly for the sensor, platform information must be processed and incorporated into the solution space to derive the required sensor parameters. The quality of the known sensor and platform parametric information directly affects the resultant operation. The less that is accurately known about the platform and sensor parameters or the existence of those parameters, also directly affect the ability to derive precise geopositions.

The frame sensor model is represented by the two pairs of equations in Equations 12 and 17. The parameters involved in Equation 12 pertain to various sources of systematic errors that affect the image coordinates. On the other hand, the parameters in Equation 17 represent the elements of the geometric model that describe the central, or perspective, projection that is the basis of frame sensor imaging. For the case of calibrated cameras, the image coordinates (x_a,y_a) in Equation 17, are in fact the coordinates (*x',y')* in Equation 12 that would have been corrected for all the systematic errors. For this case, the adjustable parameters are those six exterior orientation (EO) parameters associated with the position and attitude of the camera during the image exposure; XL, YL, ZL,, ω, ϕ, and κ appearing in Equation 17. In some cases, the camera focal length, *f*, may also be allowed to adjust, making a total of seven adjustable parameters.

For the cases where the camera is not fully metric, or when calibration is not possible/available, then a so-called self-calibration is performed during the "image adjustment" through either single image resection or multiple image triangulation. In such cases, the mathematical model is extended to include several more parameters, in addition to the six EO elements, sometimes up to a total of twenty-two parameters. These situations must be treated very carefully, as a high correlation between parameters (some approaching perfect correlation resulting in total functional dependence) can occur, leading to numerical instability resulting from singular matrices. The camera angular coverage (more problems with narrow angles), geometric arrangement of the imagery (near nadir present increased problems), character of the imaged terrain (difficulties when flat or nearly flat), etc., all contribute to high correlation. The following extended collinearity equations present a compromise for self-calibration. Note that as stated earlier, due to the relatively small influence of the k_3 term, it has been deleted from the following equations.

$$\bar{x} = x - x_0 \qquad \bar{y} = y - y_0 \qquad r^2 = \bar{x}^2 + \bar{y}^2$$

$$x = \bar{x} + \Delta x$$

$$y = \bar{y} + \Delta y$$

$$\Delta x = \frac{\bar{x}}{f}\Delta f + \bar{x}(k_1 r^2 + k_2 r^4) + p_1(2\bar{x}^2 + r^2) + p_2(2\overline{xy}) + b_1\bar{x} + b_2\bar{y} \qquad \textbf{Eq. 24}$$

$$\Delta y = \frac{\bar{y}}{f}\Delta f + \bar{y}(k_1 r^2 + k_2 r^4) + p_1(2\overline{xy}) + p_2(2\bar{y}^2 + r^2)$$

In this equation, there are nine added parameters for self-calibration: x_o, y_o, Δf, k_1, k_2, p_1, p_2, b_1, b_2. The additional symbols, to those already given and defined earlier, are:

Δf correction to the focal length, which will also accommodate a uniform scale change

b_1, b_2 two parameters to accommodate a differential scale in one axis compared to the other, and a skew between the two axes.

The full pair of equations used in estimating all the adjustable parameters are obtained by substituting the fully expanded form of (x_a, y_a) of Equation 17 into Equation 16. The total number of adjustable parameters then is fifteen, as listed in the following table.

Grouping	Number of Parameters	Parameters
a	3	X_L, Y_L, Z_L
b	3	ω, φ, κ
c	3	x_o, y_o, f or Δf
d	2	k_1, k_2
e	2	p_1, p_2
f	2	b_1, b_2
Total	15	

The way that these various sets of parameters are handled depends on the sensor type, imaging mission, and the application and use of the resulting imagery.

The degree to which the sensor adjustable parameters are known depends upon whether the imaging is performed by a fully calibrated, partially calibrated, or uncalibrated sensor. For example, if the imagery is acquired using a metric (or cartographic) aerial camera, then all the internal sensor parameters will be known from the proper laboratory calibration common to such cameras. The other extreme is when the imagery is obtained by a frame sensor, the characteristics of which are totally unknown. In such cases, some or all of the nine sensor parameters (in groups c through f above) become what are called *self-calibration parameters*. The number of parameters to be adjusted, and the identity of those parameters, depends upon the kind of sensor and the intended application.

The six parameters in groups a and b define the location and orientation of the sensor at the time of acquiring a frame image. In order to extract any positional information from such an image, numerical values for these six parameters are required. The quality of these values directly impacts the accuracy of the derived positional information. The more accurate and reliable these six parameters are the higher is the accuracy of the extracted geopositioning information. For a high-quality metric (cartographic) camera they are, in a sense, treated in an opposite manner to the sensor group of parameters. Whereas the sensor parameters are usually determined quite accurately a priori through careful calibration, the exterior (platform) parameters cannot be that well determined in advance. It is usual to carry such six elements as adjustable parameters in order to refine their initial values derived from GPS and INS. For less precise work, the GPS- and INS-provided data are usually adequate.

To summarize, for a frame image one may associate a set of adjustable parameters, the values of which are updated through a least squares adjustment. The number of such parameters can vary from as many as sixteen (including k_3) to as few as three; i.e., only ω, ϕ, κ when the GPS-provided (X_L,Y_L,Z_L) are of sufficiently high accuracy for the purpose of supporting precise geopositioning.

5.2 Covariance Matrices

In all the metric applications of imagery, the quality of the extracted information is considered as important as the information itself. This is particularly true for geopositioning applications which require high levels of accuracy and precision. The location of an object in the three-dimensional ground space is given either by its geodetic coordinates of longitude (λ), latitude (ϕ), and height (above the ellipsoid, h), or by a set of Cartesian coordinates (X,Y,Z). Although there are many ways to express the quality of the coordinates, the most fundamental is through the use of a *covariance matrix*. For example:

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} \qquad \textbf{Eq. 25}$$

in which σ_X^2, σ_Y^2, σ_Z^2 are the marginal variances of the coordinates, and σ_{XY}, σ_{XZ}, σ_{YZ} are covariances between the coordinates, which reflect the correlation between them. The practice is often to reduce these six different numbers to only two: one expressing the quality of the horizontal position and the other the quality in the vertical position. The first is called *circular error*, or CE, and the second *linear error*, or LE. Both of these can be calculated at different probability levels, CE50 for 0.5 probability, CE90 for 0.9 probability, etc. Commonly used measures, particularly by NGA under "mapping standards," are CE90 and LE90. The CE90 value is derived from the 2-by-2 sub-matrix of Σ that relates to X*, Y*, or

$$\begin{bmatrix} \sigma^2{}_X & \sigma_{XY} \\ \sigma_{XY} & \sigma^2{}_Y \end{bmatrix} \qquad \textbf{Eq. 26}$$

The LE90 is calculated from σ_Z^2. In these calculations, the correlation between the horizontal (*X,Y*) and vertical (*Z*) positions, as represented by σ_{XZ}, $\sigma_{YZ,}$ are ignored (i.e., assumed to be zero). The X,Y,Z system in these equations usually refers to the local coordinate system where Z represents elevation.

An alternative to the *covariance matrix* in Eq. 25 is the *correlation matrix*:

$$\Sigma = \begin{bmatrix} \sigma_X & r_{XY} & r_{XZ} \\ r_{XY} & \sigma_Y & r_{YZ} \\ r_{XZ} & r_{YZ} & \sigma_Z \end{bmatrix} \qquad \textbf{Eq. 27}$$

where the σ's are standard deviations, and the r's are the correlation coefficients with a defined range of ($-1 \le r \ge +1$). The covariance matrix in Eq. 25 can be constructed from Eq.27 by squaring the σ values and calculating the covariances from, for example, $\sigma_{xy} = r_{xy}\,\sigma_x\,\sigma_y$, etc.

In order to have a realistic and reliable value for the estimated covariance matrix, Σ, of the geoposition, all the quantities that enter into calculating the coordinates *X,Y,Z* must have realistic and dependable variances and covariances. These latter values present the image sensor modelers and exploiters with the most challenge. Sensor designers frequently do not provide any reasonable estimates of the expected errors associated with their sensor parameters. For well-calibrated sensors, it is usually reasonable to have the values of the needed sensor parameters as well as their quality. Note however, as stated earlier, for most practical situations, the principal point offsets and distortion parameters have such small magnitudes with regard to the other terms, it is not usual to provide covariances of these terms.

By contrast, the quality of the six exterior orientation parameters is not usually reliably known. If such parameters are carried as adjustable parameters, then it is not critical to have good prior error estimates. These prior values can be approximate since, through the adjustment process, they would be refined through rigorous error propagation associated with least squares adjustment. These updated parameter covariances are, in turn, used in a rigorous propagation to produce the final covariance matrix, Σ, associated with each object. In the metadata tables these covariance matrices will be explicitly listed as required.

The most difficulty is encountered when no adjustability is allowed and the information is based solely on the mission support data. In this case, if the input values for the quality of the parameters are either grossly in error, or non-existent, the propagated geolocation covariance matrix, Σ, can be considerably in error.

6. Frame Sensor Metadata Requirements

Geopositioning from frame sensor imagery requires pertinent metadata. Such metadata include two broad sets of parameters: interior and exterior.

The ***interior parameters*** are those which are specific to the sensor design and calibration such as focal length (*f*), location of the principal point (x_0 and y_0), and various other calibration data which allow for corrections for systematic errors within the sensor. Additionally, covariance information associated with these parameters is used in computing geoposition uncertainties and should also be provided.

The ***exterior parameters*** describe the location and orientation of the sensor with respect to the object reference coordinate system. As is clear from the details presented in the text of this formulation paper, there are many coordinate systems and sequences of rotation angles that may be involved in the various components of the collecting system. In the interest of establishing a standard, it is recommended that the location of the sensor (or more precisely, its effective perspective center, **L**) will be with respect to the Geocentric (or ECEF) reference coordinate system. The orientation of the image-frame will be provided in the form of the nine elements (m_{ij}) of the orthogonal matrix, **M**, that rotates the geocentric reference coordinate system to be parallel to the image-frame coordinate system. The elements of this matrix are functions of only three independent parameters; the most common photogrammetric standards are the three sequential rotations: ω, ϕ, and κ. The values of these angles can be readily calculated from the numerical values of the elements of **M**.

The quality of the six exterior orientation elements (location coordinates and orientation angles) is expressed by a 6×6 covariance matrix (or equivalently by a 6×6 correlation matrix with standard deviations along the main diagonal and correlation coefficients off the main diagonal). The covariance matrix will, in general, be a full matrix because it is usually calculated from several constituent covariance matrices associated with different transformations through rigorous error (or covariance) propagation.

To summarize: the required standard exterior metadata for geopositioning with a frame sensor are the coordinates (X_L, Y_L, and Z_L) of the effective perspective center in the geocentric coordinate system, the nine elements of the matrix **M** that rotates the geocentric system parallel to the image-frame coordinate system, and the 6×6 covariance matrix expressing the quality of (or uncertainties associated with) the exterior orientation elements (since this matrix is symmetric, it contains only twenty-one unique values—six variances on the main diagonal and fifteen off-diagonal covariances).

Appendix A shows how the standard exterior metadata elements might be determined and how the corresponding 6×6 covariance matrix would be developed for an example case involving five different coordinate systems (geocentric, NED, platform, sensor and frame). Other cases can similarly be addressed by the developer.

Table 1 provides the metadata for a calibrated sensor in order to derive precise geopositions. Table 2 lists the metadata for the platform, which may be required to derive some of the information appearing in Table 1.

Table 1. Frame sensor model
Metadata Parameter, Definition, Obligation, and Comments / Explanation
(Obligation: M - Mandatory, C - Conditional, O - Optional)

ID	*Parameter*	*Definition*	*Obligation/condition*	*Comments Explanation*
1	Sensor Type	Classification indicative of the characteristics of the collection device.	M	Although this paper specifically addresses non-mosaiced framing EO-IR sensors, for completeness in sensor model development this field is listed as mandatory as it is anticipated to become part of a recommended metadata "core" elements list.
2	Number of Imaging Blocks forming an Image	Total number of image blocks in a single imaging operation	C	Conditional if the digital collector is composed of more than one detector array. Each film frame image is singular.
3	Number of Columns in sensor array	C, the number of columns in the sensor array. (unitless)	C	Conditional because value can be derived from sensor array width, y-direction, divided by column spacing (d_y), if that information is available in lieu of number of columns. Note that for sensor modeling, the size of the original imaging operation must be provided; that is, the original image size before chipping (if any).
4	Number of Rows in sensor array	R, the number of rows in the sensor array. (unitless)	C	Conditional because value can be derived from sensor array width, x direction, divided by row spacing (d_x)., if that information is available in lieu of number of rows. Note that for sensor modeling, the size of the original imaging operation must be provided; that is, the original image size before chipping (if any).
5	Sensor Collection Time (POSIX TIME)	Time in micro-seconds for each image of the dataset collection based on using the Portable Operating System Interface (POSIX) where time is in integer microseconds since 1 Jan 1970 and adding required leap seconds to state UTC time.	M	Applies an IEEE standard which provides required greater significant number precision than NITF. Algorithms exist to incorporate required leap seconds to convert to UTC. Incorporates the IEEE 1003.1 Corrigendum, and Profiles PSE52 and PSE54 of IEEE 1003.13-2003, "IEEE Standard for Information Technology-Standardization Application Environment Profile-POSIX Realtime and Embedded Application Support (AEP).
6	X_L – Sensor Perspective Center Position at Sensor Collection Time (t)	X, location of the sensor in the geocentric coordinate system at time of exposure	M	Primary exterior orientation position parameter required for sensor location.
7	Y_L- Sensor Perspective Center Position at Sensor Collection Time (t)	Y location of the sensor in the geocentric coordinate system at time of exposure	M	Primary exterior orientation position parameter required for sensor location.

ID	*Parameter*	*Definition*	*Obligation/condition*	*Comments Explanation*
8	Z_L – Sensor Perspective Center Position at Sensor Collection Time (t)	Z location of the sensor in the world coordinate system at time of exposure	M	Primary exterior orientation position parameter required for sensor location.
9	Nine elements, m_{ij}, of the matrix **M**	**M** is the orthogonal matrix which rotates the geocentric coordinate system to be parallel to the image record coordinate system	M	Defined in Equation 16 with alternative methods for creation developed in Appendix A
10	$\mathbf{V}_{X_L}$	Sensor velocity in the X_L direction at Kalman filtering time stamp.	C	NED velocity vectors as out put from the Kalman filtering process translated to the sensor for each Kalman process output time stamp. Conditional if the platform velocity must be used with the IMU to Sensor Lever Arm.
11	$\mathbf{V}_{Y_L}$	Sensor velocity in the Y_L direction at Kalman filtering time stamp.	C	See V_{X_L}
12	$\mathbf{V}_{Z_L}$	Sensor velocity in the Z_L direction at Kalman filtering time stamp.	C	See V_{X_L}
13	Sensor Focal Length	*f*, lens focal length; Effective distance from optical lens to sensor element(s). A community accepted value of 999.99 indicates focal length is not available or not applicable to this sensor.	M	
14	Sensor Focal Length Flag	Value that defines if the provided focal length is a calibrated focal length, *f*, (mm); corrected effective distance from optical lens to sensor array.	M	Y(es) / N(o) or 1 /0 value that indicates Calibrated or Not-Calibrated focal length value is provided
15	Calibration Date	Date sensor was last calibrated. CCYY is the year, MM is the month (01–12), and DD is the day of the month (01 to 31).	C	Conditional on value of item 17. If Item 12 is Yes or 1 then this item is Mandatory.
16	Sensor Focal	Refinement (Δf) resulting from	M	Nominally a single value for a data set collection. Conditional on the implementation of a self-calibration

ID	***Parameter***	***Definition***	***Obligation /condition***	***Comments Explanation***
	Length Adjustment	self-calibration operation in millimeters		operation in the software.
17	Principal point off-set, x-axis	x_0, x-coordinate within the sensor array coordinate system of the foot of the perpendicular dropped from sensor perspective center onto the collection array. (mm).	C	As a coordinate, this term includes magnitude and sign (i.e., positive/negative x). Conditional when this is replaced with calibrated, or derived data.
18	Principal point off-set, y-axis	y_0, y-coordinate within the sensor array coordinate system of the foot of the perpendicular dropped from sensor perspective center onto the collection array. (mm).	C	As a coordinate, this term includes magnitude and direction (i.e., positive/negative y). Conditional when this is replace σ **Eq. 17** d with calibrated, or derived data.
19	Principal Point offset covariance data	Covariance data of principal point offsets.	O	In practice, of such small magnitude so as can be ignored.
20	Image Record Coordinate Reference Definition	Origin at sensor perspective center at a distance *f* (focal length) from the image plane; positive z-axis aligned with optical axis and pointing away from sensor and the x_r and y_r axes will be parallel to and in the same directions as the platform center of navigation axes at nadir.	M	
21	Sensor position and attitude accuracy variance data	$\sigma^2_{X_L}, \sigma^2_{Y_L}, \sigma^2_{Z_L}$ $\sigma^2_{\omega}, \sigma^2_{\phi}, \sigma^2_{\kappa}$ Variance (sigma^2) data for position (X_L,Y_L,Z_L), and attitude angles (ω, ϕ, κ)	M	Usually estimated on the basis of original data or from photogrammetric processing such as triangulation. Conditional if standard deviations are provided instead.
22	Sensor position and attitude accuracy covariancy data	$\sigma_{X_L Y_L}$, $\sigma_{X_L Z_L}$, $\sigma_{Y_L Z_L}$, $\sigma_{X_L \omega}$, $\sigma_{X_L \varphi}$, $\sigma_{X_L \kappa}$, $\sigma_{Y_L \omega}$, $\sigma_{Y_L \varphi}$, $\sigma_{Y_L \kappa}$, $\sigma_{Z_L \omega}$, $\sigma_{Z_L \varphi}$, $\sigma_{Z_L \kappa}$, $\sigma_{\omega \varphi}$,	M	

ID	*Parameter*	*Definition*	*Obligation/condition*	*Comments Explanation*
		$\sigma_{\omega\kappa}$, $\sigma_{\varphi\kappa}$		
23	Focal length accuracy variance data	σ_f^2 Variance (sigma^2) of the focal length	M	Variance of the focal parameter. Conditional if standard deviation provided instead.
24	Column Spacing	Column spacing, d_y, measured at the center of the image. Distance in the image plane between adjacent pixels within a row measured in millimeters; (00.0000 to 99.9999) or Angular center-to-center value (pitch) subtended at the perspective center, L, between adjacent pixels within a row measured in micro-radians (0000.00 to 9999.99 µ-radians). If the actual spacing (or associated units) is unknown, the default value of "0000000" will be entered.	M	
25	Row Spacing	Row spacing, d_x, measured at the center of the image. Distance in the image plane between corresponding pixels of adjacent rows measured in millimeters; (00.0000 to 99.9999) or Angular center-to-center value (pitch) subtended at the perspective center, L, between corresponding pixels of adjacent rows measured in microradians.(0000.00 to 9999.99). If the actual spacing (or associated units) is unknown, the default value of "0000000" will	M	

ID	*Parameter*	*Definition*	*Obligation/condition*	*Comments Explanation*
		be entered.		
26	Corrections for various distortions in line (a_1,b_1,c_1) and sample (a_2,b_2,c_2) coordinates	Collectively represents 2 scales, rotation, skew, and 2 shifts applied in an affine transformation of the line and sample image coordinates	M	The six parameters collectively correct for various systematic image distortions.
27	Distortion correction (a_1,b_1,c_1,a_2,b_2,c_2) covariance data	A 6 by 6 covariance matrix reflecting the quality of the six distortion correction values	O	In practice, of such small magnitude, so as can be ignored.
28	Column axis offset	C_ℓ, linear translation from the image upper-left corner pixel to the collection array origin (mm), s-axis	C	Conditional, as can be derived if other physical properties are known; number of rows and row spacing.
29	Row axis offset	C_s, linear translation from the image upper-left corner pixel to the to collection array origin (mm), ℓ-axis	C	Conditional, as can be derived if other physical properties are known; number of columns and column spacing.
30	Radial lens distortion coefficients	k_0 (mm^0) k_1 (mm^-2), k_2 (mm^-4), k_3 (mm^-6), radial lens distortion coefficients	C	Conditional when replaced with calibration, or derived data.
31	Radial lens distortion (k_0, k_1,k_2,k_3) covariance data	Covariance data of radial lens distortion coefficients.	O	In practice, of such small magnitude, so as can be ignored.
32	Decentering lens distortion coefficients	$p_1(mm^{-1})$, $p_2(mm^{-1})$	C	Conditional when replaced with calibration, or derived data.
33	Decentering lens distortion (p_1,p_2) covariance data	Covariance data of decentering lens distortion coefficients.	O	In practice, of such small magnitude, so as can be ignored.
34	Atmospheric correction (Δd) by data layer	Correction to account for bending of the image ray path as a result of atmospheric effects	C	Adjustment to compensate for the bending in the image ray path from object to image due to atmospheric effects. Multiple data layers can be defined so the parameter has an index of I= 1, …n

ID	***Parameter***	***Definition***	***Obligation/condition***	***Comments Explanation***
35	Atmospheric correction data layer top height	Upper boundary altitude value for data layer I	C	Sets the upper bound for the specific atmospheric correction value for data layer I
36	Atmospheric correction data layer bottom height	Lower boundary altitude value for data layer I	C	Sets the lower bound for the specific atmospheric corrections value for data layer I
37	Atmospheric correction algorithm name	Name of algorithm used to compute data layer I correction	C	Defines the specific algorithm used in the computation
38	Atmospheric Correction algorithm version	Version label for the algorithm used to compute data layer I correction	C	Defines the specific version of the algorithm used in the computation

Table 2. Collection platform parameters

(Requirement: M - Mandatory, C - Conditional, O – Optional, TBR – To be resolved)

ID	*Parameter*	*Definition*	*Rqmt*	*Comments*
39	Ephemeris Flag	Flag used to indicate the source of (orbit/ Flight) determination) ephemeris data used for this data set	M	All sensors are expected to employ GPS. There should be no difference in ephemeris data whether it is from an airborne or satellite platform. The GPS data is derived as ECEF X, Y, and Z. Requirement should be for COLLECT-TIME = actual real time or REFINED = refined real time ephemeris, not PREDICTED. Since the platform is obtaining its positional information at a different time sequence than the sensor is acquiring data, the platform positional information needs to be interpolated to the sensor image time. This is often accomplished with Kalman filtering or quaternions. Normally a set of seven observations that bracket the sensor data acquisition time is used.
40	Platform Time	Time at which data was collected.	C	Provides data to correlate platform location to sensor acquisition. **Mandatory** if location not simultaneously collected with image data, to provide necessary location of the sensor/platform/Earth reference coordinate system to allow correct interpolation at image acquisition time.
41	Platform geo-location	The position of the platform given as X, Y, and Z Ephemeris Vectors in ECEF coordinates (meters).	C	Not required for image-to-ground calculations if sensor location data available directly. Center of navigation defined with respect to the local NED coordinate frame using offsets, and then related to an ECEF reference.
42	Platform true heading at image time	Platform heading relative to true north. (positive from north to east) (degrees)	C	Conditional if sensor position and rotation data not available directly when given within an absolute reference frame. Alternatively, true heading not required if platform yaw is given,
43	Platform pitch	Rotation about platform local y-axis (Y_p), positive nose-up; 0.0 = platform z-axis (Z_p) aligned to Nadir, limited to values between +/-90 degrees. (degrees)	C	Conditional if sensor position and rotation data not available directly when given within an absolute reference frame.
44	Platform roll	Rotation about platform local x-axis (X_p). Positive port wing up. (degrees)	C	Conditional if sensor position and rotation data not available directly when given within an absolute reference frame.
45	GPS Lever arm offset	Vectors from GPS to INS described in either x, y, z components or by magnitude and two rotations and velocity.	C	Conditional on sensor geolocation at image exposure time being provided. If sensor geolocation is provided based on INS processing, this lever arm is mandatory to establish platform GPS to INS geolocation and velocity.
46	INS Lever arm offset	Vectors from INS to Sensor described in either x, y, z components or by magnitude and two rotations, velocity, and platform attitude	C	Conditional on sensor location and attitude at image exposure time being provided directly. If sensor geolocation and attitude is provided based on INS processing, this level arm is mandatory to establish sensor's six elements of location and orientation.

ID	*Parameter*	*Definition*	*Rqmt*	*Comments*
47	X-Component of the Sensor Offset Vector	The X-axis component, measured in the platform coordinate system, from the origin of the platform coordinate system to the origin of the sensor coordinate system, i.e. the perspective center L.	C	Offset vector describes the position of the sensor perspective center relative to the platform in the platform coordinate system. Conditional for the case where a sensor does not provide position information directly referenced to, say, an ECEF system.
48	Y-Component of the Sensor Offset Vector	The Y-axis component, measured in the platform coordinate system, from the origin of the platform coordinate system to the origin of the sensor coordinate system, i.e. the perspective center L.	C	See X-Component of the Sensor Offset Vector
49	Z-Component of the Sensor Offset Vector	The Z-axis component, measured in the platform coordinate system, from the origin of the platform coordinate system to the origin of the sensor coordinate system, i.e. the perspective center L.	C	See X-Component of the Sensor Offset Vector
50	Roll: Sensor Rotation about the translated platform X_p -axis	The rotation of the sensor in the yz-plane of the sensor reference frame; measured as positive when positive y-axis rotates directly towards the positive z-axis.	M	Reference Figure 1. If the sensor is fixed in position and its axes are perfectly aligned with the platform axes, then platform attitude is sensor attitude. The INS to Sensor Vector identifies angular adjustments to platform attitude, which then defines a static platform to sensor attitude regardless of platform attitude, and effectively translates platform attitude to the sensor coordinate system origin. Thus sensor x_s is aligned at with the platform X_p axis and the sensor y_s z_s plane is aligned with the platform Y_p Z_p plane. In flight processing of the platform attitude data is then translated to the sensor coordinate origin and any local sensor rotations are applied to these platform attitude values to define sensor attitude at time of exposure. Sensor roll is angular position of the sensor optical axis, measured about the platform roll axis (Xp). Measured positive from the positive pitch axis vector (+Yp) toward the positive yaw axis vector (+Zp) (clockwise looking in the +Xp direction). Reference may be made to gimbal mounting or to platform reference system; but must be specified. If the sensor is gimbal mounted and can be directed by rotations to an alternate viewing position, then the local gimbal rotations shall be applied to provide sensor attitude at time of exposure. If the rotation angles are gimbal mounting angles, the photogrammetric development transforms them into the required sequential Euler angles.

ID	***Parameter***	***Definition***	***Rqmt***	***Comments***
				An alternative method to determining sensor orientation angles, as described in this item, is to employ quaternions described in item 15.
51	Pitch: Sensor Rotation about the translated Platform Y_p-axis	Rotation around the once rotated sensor y'-axis" defined as the rotation of the sensor in the once rotated x'z'-plane of the sensor reference frame; measured as positive when the positive z'-axis rotates directly towards the positive x'-axis.	M	See Sensor Rotation about platform X_p-axis translated to sensor coordinate system origin. Sensor pitch is the angular position of the sensor optical axis, measured about the once rotated platform pitch axis (Yp).. Measured positive from the positive yaw axis vector (+Zp) toward the positive roll axis vector (+Xp) (clockwise looking in the +Yp direction). An alternative method to determining sensor orientation angles, as described in this item, is to employ quaternions described in item 15.
52	Yaw: Sensor Rotation about translated Platform Zp-axis	Rotation around the sensor twice rotated z"-axis defined as the rotation of the sensor in the x"y"-plane of the sensor reference frame; measured as positive when the positive x"-axis rotates directly towards the positive y"-axis.	M	See Sensor Rotation about platform X_p-axis translated to sensor coordinate system origin. Sensor yaw is the angular position of the sensor optical axis (line of sight), measured about the twice rotated platform yaw axis (Zp). It is the angle from the positive roll axis vector (+Xp) to the projection of the sensor optical axis onto the Xp-Yp plane. Measured positive from the positive roll axis vector (+Xp) toward the positive pitch axis vector (+Yp) (clockwise looking in the +Zp direction). An alternative method to determining sensor orientation angles, as described in this item, is to employ quaternions described in item 15.
53	Quaternions of Attitude Reference Point	A set of four quaternions (Q1, Q2, Q3, and Q4) derived from sensor ephemeris data that provide sensor attitude information required to process the sensor rigorous math model to perform geolocation and mensuration.	C	With ephemeris data for the platform, the derivation of the set of four Quaternions (Q1, Q2, Q3, and Q4) define sensor Attitude Reference Points in the ECEF coordinate system. Conditional only if platform ephemeris information is not available for airborne platform sensors. Mandatory for satellite platform sensors.

Table 3
Hierarchical Order of Metadata Elements for Precise Geopositioning

The optimal situation is that for each:
Image collection time,

the following data set of parameters identified in Section 5.1:

Sensor X, Y, Z Position (X_C, Y_C, Z_C ,
Orientation Data (nine elements of the rotation matrix **M**)
Principal point offset, x-axis(x_o)
Principal point offset, y-axis (y_o)
Focal Length or focal length correction (Δf)
Lens radial distortion coefficients (k_1, k_2)
Decentering lens correction coefficients (p_1, p_2)
Correction coefficients (a_1, b_1, c_1 for line and a_2, b_2, c_2 for sample coordinates)
Row spacing and column spacing (d_x and d_y)
are provided.

In addition, the associated M(andatory) variance and covariance data:

Sensor position and attitude accuracy variance and covariance data (21 elements of the covariance matrix described in Appendix A (M)
Focal length variance data (M)
Principal point offset covariance data (O)
Lens radial distortion covariance data (O)
Decentering lens correction covariance data (O)
Scale and skew correction covariance data (O)

need to be directly available for that frame sensor image.

In order to correctly define the sensor, the row element and column element spacing values are also (M(andatory)) required.

If these items are not available then they must be created from other platform or sensor data. For example, sensor position and sensor orientation can be developed by adjusting INS Platform Position and Attitude with the INS to Sensor Lever Arm data (Appendix A provides a derivation example).

Appendix A

Equations to Map Individual Covariances to Full Covariance Matrix of Standard Six Frame EO Parameters

4. Introduction:

The objective of this document is to provide the equations required to map covariance matrices of the individual error components of a frame imaging system to a full 6 by 6 covariance matrix associated with a standard frame camera sensor model. The top left 3 by 3 is the covariance matrix associated with position and lower right 3 by 3 is associated with the standard three angles, $\delta\omega$, $\delta\varphi$, $\delta\kappa$, representing attitude errors about the x, y, z axes, respectively, of a frame coordinate system. These three angles represent the collective effects of several other orientation angles on which will be elaborated in the following sections.

5. Coordinate Systems:

Figures 1 and 2 illustrate the coordinate systems involved in a typical airborne optical frame imaging system, namely Geocentric (g), North-East-Down, or NED (n), Platform (p), Sensor (s), and Record (r). Such a configuration is consistent with the 2009 versions of SENSRB and EG0801 documents. The NED and platform systems have the same origin at the center of navigation. When all platform angles (heading, pitch, and roll) are zeros, these two systems are coincident. Occasionally, a local object coordinate system, East-North-Up, or ENU, is used in place of the Geocentric system. However, for this development the Geocentric is selected because of being the desired standard.

As shown in Figure 2, the camera perspective center location is a function of the GPS antenna location, the base (aka lever arm) vector from the origin of the GPS antenna to the perspective center, and the platform attitude with respect to the GCS.

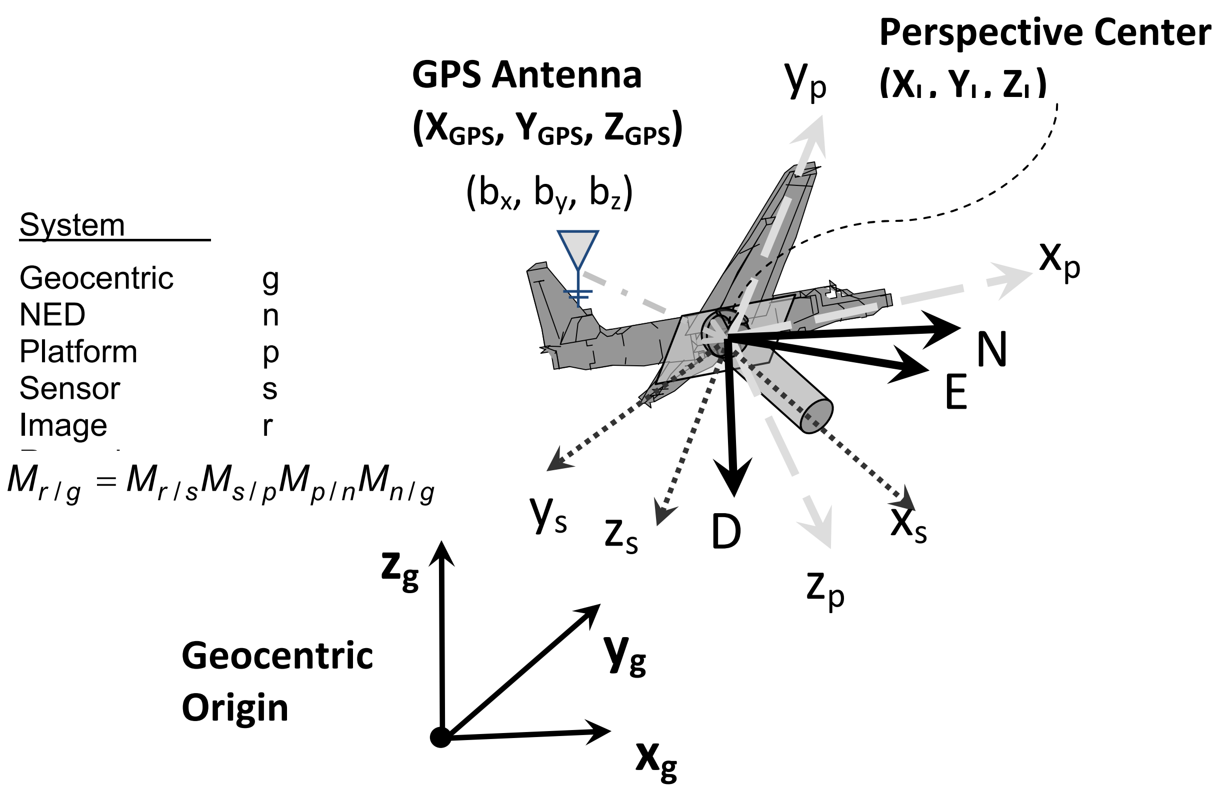

Figure 1. Coordinate Systems overlaid on aircraft

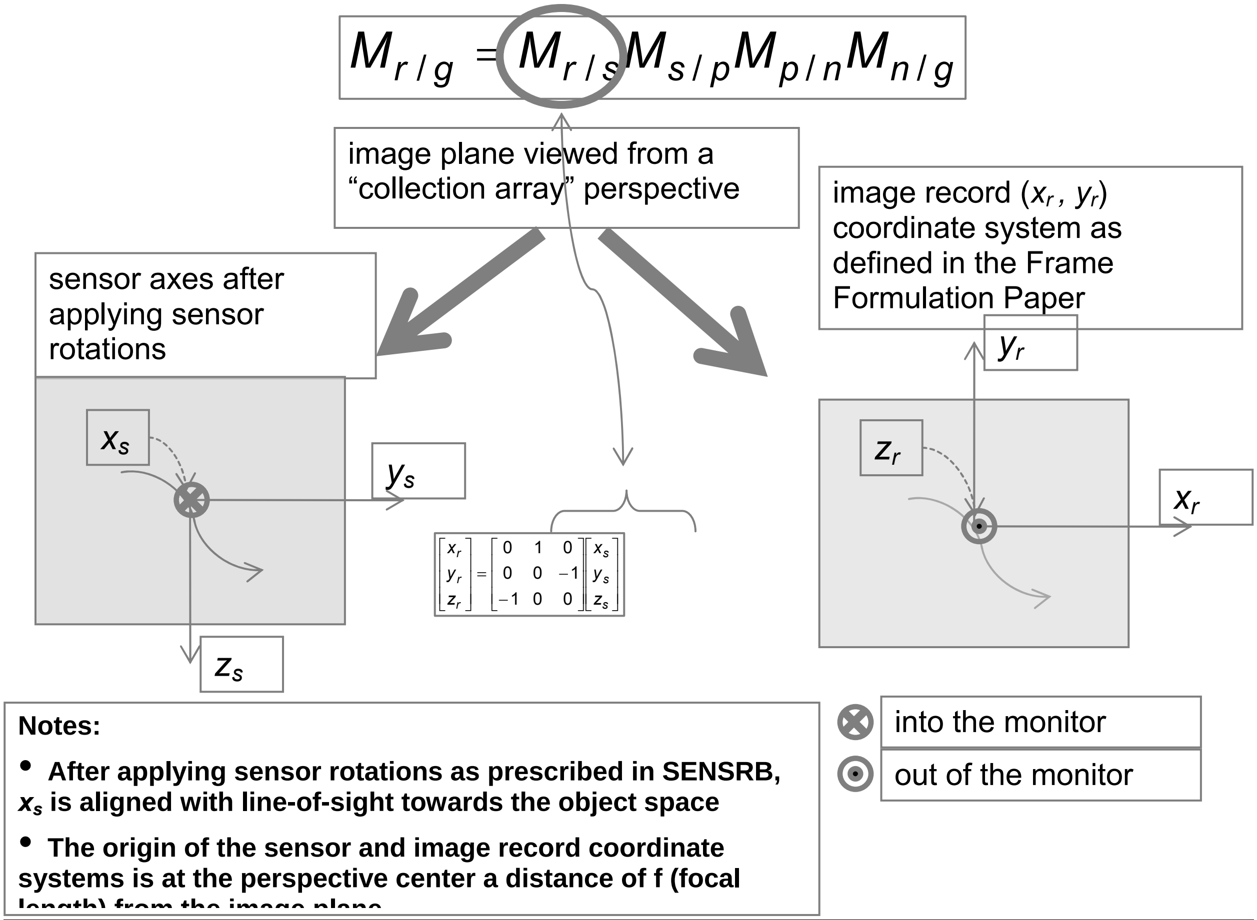

Figure 2. Sensor and Record Coordinate Systems as viewed on a monitor

6. Projection Model:

The following derivation will, in general, use the matrix notation $M_{b/a}$ to designate an orthogonal matrix that rotates coordinate system "a" until it is parallel with coordinate system "b". The collinearity equation can be written as follows:

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = kM \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} = kM_{r/s} M_{s/p} M_{p/n} M_{n/g} \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix} \qquad \textbf{Eq. A.1}$$

Where x,y are image coordinates (shifted to the principal point and corrected for all systematic errors) in a Record Coordinate System (RCS); f is the focal length; k is a unique scale factor per ground point; X,Y,Z are ground coordinates in a Geocentric, or Earth-Centered-Earth-Fixed (ECEF), Coordinate System (GCS); and X_L, Y_L, Z_L are the coordinates of the camera perspective center in the GCS; $M_{r/s}$ is the orthogonal rotation matrix that aligns the sensor coordinate system to the image record coordinate system; $M_{s/p}$ aligns the platform to the sensor coordinate system; $M_{p/n}$ aligns the NED to the platform coordinate system; and $M_{n/g}$ aligns the Geocentric to the NED coordinate system.

As shown in Figure 1, the camera perspective center location is a function of the GPS antenna location, the base (aka lever arm) vector from the origin of the GPS antenna to the perspective center, and the platform attitude with respect to the GCS. Note that we selected, as an example, the case where the perspective center (L) is also at the origin of the NED (and the platform) coordinate system or center of navigation. In Figure 1 of Section 2.1 of the Frame Formulation Paper, the general case is shown where there is another offset vector from the platform origin to the image record perspective center. The coordinates of the perspective center in the GCS are given by:

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} X_{GPS} \\ Y_{GPS} \\ Z_{GPS} \end{bmatrix} + M_{n/g}{}^T M_{p/n}{}^T \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix} \qquad \textbf{Eq. A.2}$$

where bx, by, bz are the components of the base vector measured in the Platform Coordinate System. By substituting Eq. A.2 into Eq. A.1, we obtain:

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = kM_{r/s}M_{s/p}M_{p/n}M_{n/g}\left(\begin{bmatrix} X - X_{GPS} \\ Y - Y_{GPS} \\ Z - Z_{GPS} \end{bmatrix} - M_{n/g}{}^T M_{p/n}{}^T \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix} \right) \qquad \textbf{Eq. A.3}$$

4. Stochastic Model:

In order to establish the covariance mapping equations, we must define the stochastic models for the standard frame camera sensor model and then for the example frame imaging system.

The stochastic model for the standard frame involves re-formulating Eq. A.1 such that it isolates the random variation to six adjustable parameters with zero expected values (these six parameters correspond to the standard six Exterior Orientation elements of a frame image); the middle part of Eq. A.1 is re-written as follows:

$\underset{3\times1}{a} = k\underset{3\times3}{(\delta M)}\underset{3\times3}{M}\underset{3\times1}{A}$**,** which in expanded form becomes:

$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = k(\delta M)M = k\begin{bmatrix} 1 & \delta\kappa & -\delta\varphi \\ -\delta\kappa & 1 & \delta\omega \\ \delta\varphi & -\delta\omega & 1 \end{bmatrix} M \begin{bmatrix} X - (X_L + \delta X_L) \\ Y - (Y_L + \delta Y_L) \\ Z - (Z_L + \delta Z_L) \end{bmatrix} \qquad \textbf{Eq. A.4}$$

in which $M = M_{r/s}M_{s/p}M_{p/n}M_{n/g}$. The combined effect of all the component matrices, as represented by M, is the three "generic" sequential angles, ω, φ, κ, which can be extracted from the elements of M, if needed. The attitude errors, all of which have zero expected value, are then manifested by the three terms, δω, δφ, δκ.

The stochastic model for the example frame imaging system involves re-formulating Eq. A.3 such that it isolates each error component as follows:

$$\begin{bmatrix} x \\ y \\ -\ f \end{bmatrix} = k \underset{3\times 3}{T} \underset{3\times 1}{u} \qquad \textbf{Eq. A.5}$$

where T and u are a temporary matrix and vector, respectively, used to break the long equation into two separate pieces as follows:

$$T = M_{f/s}\begin{bmatrix} 1 & 0 & -\delta R_p \\ 0 & 1 & 0 \\ \delta R_p & 0 & 1 \end{bmatrix} M_{2R} \begin{bmatrix} 1 & \delta R_h & 0 \\ -\delta R_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} M_{3R} \begin{bmatrix} 1 & \delta I_h & -\delta I_p \\ -\delta I_h & 1 & \delta I_r \\ \delta I_p & -\delta I_r & 1 \end{bmatrix} M_{p/n} M_{n/g} \qquad \textbf{Eq. A.6}$$

where M_{2R} and M_{3R} are the rotation matrices that are a function of gimbal resolver measurements in sensor pitch and heading, respectively; i.e., $M_{c/p} = M_{2R}M_{3R}$. The R stands for resolver and the 2 and 3 correspond to the axis of rotation, i.e. about the current Y and Z axes, respectively.

$$u = \left(\begin{bmatrix} X - (X_{GPS} + \delta X_G) \\ Y - (Y_{GPS} + \delta Y_G) \\ Z - (Z_{GPS} + \delta Z_G) \end{bmatrix} - M_{n/g}{}^T M_{p/n}{}^T \begin{bmatrix} 1 & \delta I_h & -\delta I_p \\ -\delta I_h & 1 & \delta I_r \\ \delta I_p & -\delta I_r & 1 \end{bmatrix}^T \begin{bmatrix} b_X + \delta b_X \\ b_Y + \delta b_Y \\ b_Z + \delta b_Z \end{bmatrix} \right) \qquad \textbf{Eq. A.7}$$

The terms $\delta X_G, \delta Y_G, \delta Z_G$ are error terms in the GPS platform position, and δb_X, δb_Y, δb_Z are error terms associated with the offset base; both sets are additive. (All six error terms have zero mean expectations, and two finite 3 by 3 error covariance matrices, Σ_{GG} and Σ_{BB}, respectively). Errors in the platform orientation are matrix multiplicative and are given by the symbols $\delta I_h, \delta I_p, \delta I_r$, in Eq. A.7. (The three error terms have zero expectations and a 3 by 3 covariance matrix, Σ_{II}). The error contributors can be grouped into vectors of random variables, GPS (G), INS (I), base (B), and gimbal resolver (R), as follows:

$$\underset{3\times 1}{l_G} = \begin{bmatrix} \delta X_G & \delta Y_G & \delta Y_G \end{bmatrix}^T \qquad \textbf{Eq. A.8}$$

$$\underset{3\times 1}{l_B} = \begin{bmatrix} \delta b_X & \delta b_Y & \delta b_Z \end{bmatrix}^T \qquad \textbf{Eq. A.9}$$

$$\underset{3\times 1}{l_I} = \begin{bmatrix} \delta I_r & \delta I_p & \delta I_h \end{bmatrix}^T \qquad \textbf{Eq. A.10}$$

$$\underset{2\times 1}{l_R} = \begin{bmatrix} \delta R_p & \delta R_h \end{bmatrix}^T \qquad \textbf{Eq. A.11}$$

$$\underset{11\times 1}{l} = \begin{bmatrix} \underset{1\times 3}{l_G}^T & \underset{1\times 3}{l_B}^T & \underset{1\times 3}{l_I}^T & \underset{1\times 2}{l_R}^T \end{bmatrix}^T$$ **Eq. A.12**

Note in Eq. A.6 that the INS angle errors can be modeled in a combined matrix since the navigator performs calculations in an inertial system based on IMU measurements and Kalman Filtering; hence the output of such calculations is an attitude error covariance referenced to the current platform coordinate system. However, the resolver angle errors need to be modeled in separate error covariance matrices since each angle measurement is made sequentially.

At a high level, we can summarize the covariance propagation required to map from example imaging system to standard frame system as follows:

$$\begin{aligned} E &= f_1(P, A) = f_1(l_G, l_B, l_I, l_R) = f_1(l) \\ P &= f_2(l_G, l_I, l_B) \\ A &= f_3(l_I, l_R) \end{aligned}$$ **Eq. A.13**

where E, P, A represent exterior orientation, position, and attitude, respectively, of the standard record system; and f1, f2, and f3 symbolically represent functions. Since the INS components appear in both P and A, clearly the covariance propagation will result in a full 6 by 6 covariance matrix, i.e. representing correlation between position and attitude.

We can now apply the general error propagation equation to the first line of Eq. A.8 as follows:

$$\underset{6\times 6}{\Sigma_{EE}} = \underset{6\times 15}{J_{El'}} \underset{15\times 15}{\Sigma_{l'l'}} \underset{15\times 6}{J_{El'}}^T = \begin{bmatrix} \underset{3\times 3}{\Sigma_{PP}} & \underset{3\times 3}{\Sigma_{PA}} \\ \underset{3\times 3}{\Sigma_{PA}}^T & \underset{3\times 3}{\Sigma_{AA}} \end{bmatrix}$$ **Eq. A.14**

where Σ_{PP} is the position covariance matrix, Σ_{AA} is the attitude covariance matrix, and Σ_{PA} is the cross-covariance matrix between position and attitude.

Note that the 15 by 1 vector *l'* is referenced in this equation instead of the 11 by 1 vector *l*. We need to introduce fictitious observations with zero values and zero errors in order to facilitate the covariance propagation. When a gimbal resolver measures an angle, e.g. in heading, it is known that the rotation and associated precision of the angles in pitch and roll will be zeros; hence the placeholders associated with Rp and Rr were zeroed out in the second expanded matrix of Eq. A.6. Similarly when the gimbal resolver measures the pitch, it is known that the rotation and associated precision of the angles in heading and roll will be zeros; hence the placeholders associated with Rh and Rr are zeroed out in the first expanded matrix of Eq. A.6).

We can expand the Jacobian matrix in Eq. A.9 as follows:

$$\underset{6\times 15}{J_{El'}} = \begin{bmatrix} \underset{3\times 3}{J_{PG}} & \underset{3\times 3}{J_{PB}} & \underset{3\times 3}{J_{PI}} & \underset{3\times 6}{0} \\ \underset{3\times 3}{0} & \underset{3\times 3}{0} & \underset{3\times 3}{J_{AI}} & \underset{3\times 6}{J_{AR}} \end{bmatrix}$$ **Eq. A.15**

where the Jacobian sub-components corresponding to position can be obtained by referencing Eq.A.2 as follows:

$$J_{PG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **Eq. A.16**

$$J_{PB} = M_{n/g}{}^{T} M_{p/n}{}^{T}$$ **Eq. A.17**

$$J_{PI} = \begin{bmatrix} J_{PIr} & J_{PIp} & J_{PIh} \end{bmatrix}$$ **Eq. A.18**

$$J_{PIr} = M_{n/g}{}^{T} M_{p/n}{}^{T} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix}$$ **Eq. A.19**

$$J_{PIp} = M_{n/g}{}^{T} M_{p/n}{}^{T} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix}$$ **Eq. A.20**

$$J_{PIh} = M_{n/g}{}^{T} M_{p/n}{}^{T} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix}$$ **Eq. A.21**

and the Jacobian sub-components corresponding to attitude can be obtained by referencing Equations A.4 , A.5, A.6 and A.7 as follows:

$$J_{AI} = M_{r/s} M_{s/p}$$ **Eq. A.22**

$$J_{AR} = \begin{bmatrix} \underset{3\times3}{M_{r/s}} & \underset{3\times3}{M_{r/s}} M_{2R} \end{bmatrix}$$ **Eq. A.23**

The covariance matrix for the 15 by 1 vector *l'* can be constructed as follows:

$$\underset{15\times15}{\Sigma_{l'l'}} = \begin{bmatrix} \underset{3\times3}{\Sigma_{GG}} & \underset{3\times3}{0} & \underset{3\times3}{0} & \underset{3\times6}{0} \\ & \underset{3\times3}{\Sigma_{BB}} & \underset{3\times3}{0} & \underset{3\times6}{0} \\ & & \underset{3\times3}{\Sigma_{II}} & \underset{3\times6}{0} \\ sym & & & \underset{6\times6}{\Sigma_{RR}} \end{bmatrix}$$ **Eq. A.24**

The Σ_{GG}, Σ_{BB}, and Σ_{II} matrices are in general full 3 by 3 covariance matrices provided in the image metadata. The 6 by 6 covariance matrix Σ_{RR} would be constructed as a function of the elements of a full 2 by 2 covariance matrix of resolver angles as follows:

$$\Sigma_{RR} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \sigma_{Rp}^{2} & 0 & 0 & 0 & \sigma_{RpRh} \\ & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \\ sym & & & & & \sigma_{Rh}^{2} \end{bmatrix}$$ **Eq. A.25**

where σ_{Rp}^{2}, σ_{Rh}^{2}, σ_{RpRh} are the variance of pitch resolver measurement, variance of heading resolver measurement, and covariance between pitch and heading resolver measurements, respectively.

5. Matlab Example:

Synthetic Frame Image:
Focal length = 152mm
Flying height = 1000 m AGL = 1000 m HAE
4 check points, one at each corner of a 100mm by 100mm frame
Base (GPS to perspective center lever arm components, meters) = 15, 11, -12
Platform heading, pitch, roll (deg) = 40, -15, 13
Sensor heading, pitch (deg) = 45, -50 (note: -90 deg is nadir when platform is level)
Input Precisions:
Image coordinate sigmas = 0.015mm
Check point height sigmas = 1 m

GPS covariance (meters squared): $\Sigma_{GG} = \begin{bmatrix} 4 & 1 & 1 \\ & 4 & 1 \\ sym & & 9 \end{bmatrix}$

Base covariance (meters squared): $\Sigma_{BB} = \begin{bmatrix} 1 & 0.5 & 0.5 \\ & 1 & 0.5 \\ sym & & 1 \end{bmatrix}$

INS covariance (radians squared): $\Sigma_{II} = \begin{bmatrix} 0.0002 & 0.00008 & 0.00005 \\ & 0.0001 & 0.00006 \\ sym & & 0.0001 \end{bmatrix}$

Gimbal resolver covariance (radians squared): $\Sigma_{resolver} = \begin{bmatrix} 0.00005 & 0.00002 \\ sym & 0.00006 \end{bmatrix}$

Note that the magnitudes of some of the numbers are unrealistic, e.g. the base vector and the existence of correlation between resolver angles, but did not want to assume diagonal matrices in order to fully test the theory.

Note that the magnitudes of the elevation angles (90 degrees minus the off-nadir angle) for check points (CP) 1 through 4 were 60, 32, 57, and 30 degrees, respectively.

For these four check points, the following output provides a comparison of the 3 by 3 ground coordinate covariance matrix derived using the covariance mapping technique outlined above versus that derived using standard error propagation of all components directly. Then, the results are shown for the case assuming a block diagonal of 3 by 3 sub-matrices, i.e. ignoring correlation between position and attitude.

5.1. Results using full 6 by 6 versus direct error propagation:

CP1.	220.618170037311	-40.9940694361992	0.271504504153547
	-40.9940694361992	352.766249870072	-0.540769287713719
	0.271504504153547	-0.540769287713719	1.00010607734182
	220.618170037241	-40.9940694361544	0.271504504153541
	-40.9940694361544	352.766249869869	-0.540769287713701
	0.271504504153541	-0.540769287713701	1.00010607734182

CP2.	1208.92820880467	184.733458783294	-1.53820779310716
	184.733458783294	469.045473082101	-1.06294413660742
	-1.53820779310723	-1.06294413660742	1.0008688509752
	1208.92820880457	184.733458783221	-1.53820779310712
	184.733458783221	469.045473082005	-1.06294413660738
	-1.5382077931072	-1.06294413660739	1.0008688509752
CP3.	190.677807214114	-128.486508018024	0.190176456243646
	-128.486508018024	352.427637189316	0.647247263698036
	0.190176456243646	0.647247263698035	1.00013330292768
	190.677807214043	-128.486508017952	0.190176456243651
	-128.486508017952	352.427637189112	0.647247263698018
	0.190176456243651	0.647247263698018	1.00013330292768
CP4.	2359.69170296035	-1048.46783817954	-2.14793032601751
	-1048.46783817954	1076.8600142002	1.28693774835313
	-2.14793032601759	1.28693774835317	1.00113771021422
	2359.69170296007	-1048.46783817926	-2.14793032601753
	-1048.46783817926	1076.86001420002	1.28693774835314
	-2.14793032601748	1.28693774835308	1.00113771021422

5.2. *Results* assuming a block diagonal of 3 by 3 sub-matrices:

CP1.	221.778172948054	-42.0725039341913	0.271634601261149
	-42.0725039341913	363.131015549378	-0.541643820765941
	0.271634601261149	-0.541643820765941	1.00010615259039
	220.618170037241	-40.9940694361544	0.271504504153541
	-40.9940694361544	352.766249869869	-0.540769287713701
	0.271504504153541	-0.540769287713701	1.00010607734182
CP2	1199.87542860969	186.91711583948	-1.53671337553236
	186.917115839481	467.609993108896	-1.06316837872444
	-1.53671337553241	-1.06316837872445	1.0008685838565
	1208.92820880457	184.733458783221	-1.53820779310712
	184.733458783221	469.045473082005	-1.06294413660738
	-1.5382077931072	-1.06294413660739	1.0008688509752
CP3.	192.886019869503	-131.510885686274	0.189947663645153
	-131.510885686274	361.577818631608	0.648040083681862
	0.189947663645153	0.648040083681862	1.00013337253685
	190.677807214043	-128.486508017952	0.190176456243651
	-128.486508017952	352.427637189112	0.647247263698018
	0.190176456243651	0.647247263698018	1.00013330292768

CP4.	2351.97152640524	-1050.11540157101	-2.14637694948784
	-1050.11540157101	1072.41412797991	1.28669965521726
-	2.146376949488	1.28669965521725	1.00113731841628
	2359.69170296007	-1048.46783817926	-2.14793032601753
	-1048.46783817926	1076.86001420002	1.28693774835314
	-2.14793032601748	1.28693774835308	1.00113771021422

5.3. Observations:

As shown in the overall and the components results, the covariance mapping technique provides essentially equivalent results to direct error propagation of the individual components.

The differences appear to be due to only round-off errors.

The differences between the two methods are significant (beyond round-off errors) in the case where the 6 by 6 exterior orientation covariance matrix is treated as a block diagonal (two 3 by 3 blocks, i.e. ignoring the cross covariance matrix Σ_{PA}, in Eq. 9.

While the approach in this appendix addressed the case where the GPS antenna is offset from the camera perspective center, it can be extended to handle other cases, e.g. where the origins of the platform and gimbal systems do not coincide with the perspective center.

The covariance mapping technique presented in this appendix provides the additional benefit of, as a by-product, defining a reduced set of adjustable parameters for a sensor model in a standard reference frame.